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The New President of the National Council of Teachers of Mathematics
F. LYNWOOD WREN

THE MATHEMATICS TEACHER

Volume XXXVII Number 4

Edited by William David Reeve

The Progressive Nature of Learning in Mathematics[1]

By WILLIAM A. BROWNELL
Duke University

THERE IS A close relation between our teaching procedures and our conception of the learning process. As the first step in teaching we more or less carefully determine our objectives. Then, to help our pupils attain these objectives, we select our explanations, our types of practice materials, and our applications largely in the light of our theory of learning. It follows therefore that his view of learning is a critical part of every teacher's professional equipment.

LEARNING AS CONNECTION-FORMING

The conception of learning which has prevailed in American education for more than a quarter-century—less so now than formerly—is part of the psychological system known as connectionism. According to this view, all learning consists in the addition, the elimination, and the organization of connections—this, and nothing else. These connections are formed, or broken, or organized, between situations and responses. The process of teaching, then, comprises the following steps:

1. Identify for the learner the stimuli (or the situation) to which he is to react,
2. Identify the reaction (or response) which he is to make,
3. Have the learner make this response to the situation under conditions which reward success and which punish failure,
4. Repeat step (3) until the connection has been firmly established.

The connectionistic view of learning has not, in my opinion, been very helpful to teachers.[2] Advocates of connectionism could hardly accept my evaluation. If they were to concede that their view of learning has not always produced the best

[1] Paper read before the Regional Meeting of *The National Council of Teachers of Mathematics* in Detroit, February 19, 1944.

[2] I have never been able to find much that is *wrong* (demonstrably unsound) in connectionism. Most of the direct attacks, theoretical and experimental, seem to me to be rather futile: they have failed to show that learning is anything other than the formation of connections (if the term "connection" be interpreted as broadly as connectionists interpret it). As a matter of fact, I suspect that, neurologically at least, something very much like the processes suggested by connectionists actually occurs in learning (this, in spite of the research of Lashley and others). Objection here is raised to connectionism purely on the ground, as stated above, that it has not been helpful in the practical business of educating children. Perhaps an analogy is in order. The layman reads that all matter is reducible, according to modern physics, to electrons, neutrons, etc., in a word, to nonmatter. He can accept this view of matter as a fact. At the same time, acceptance of this view does not in any way affect the manner in which he deals with matter, however essential this view is to the work of the research physicist.

results, they would insist that the deficiency lies, not in the theory, but in the user. They would say, as many of them have said, that the theory is sound and adequate, but that it has been misinterpreted and misapplied.[3]

In this paper I want to consider with you four weaknesses in classroom instruction in mathematics. (They are by no means confined to the teaching of mathematics.) These weaknesses persist after thirty years and more of connectionism. Whether they are still with us because of the connectionistic view of learning or in spite of it, it is impossible to say. It is however possible to show how this view of learning seems to support, even to demand, the malpractices which I shall discuss. It is for this reason that the connectionistic view of learning will come in for unfavorable comment.

I am well aware that I am talking, not to professional psychologists, but to teachers of mathematics. Indeed, it is precisely because I *am* talking to teachers of mathematics that I speak as I shall. Perhaps no other subject in the curriculum so much as mathematics has suffered from the general application or the general misapplication of connectionistic theory. But my remarks will not all be negative. On the contrary, as the subject of the paper implies, I shall try to substitute positive notions and shall try to sketch a different view of learning which may be more useful to teachers.

Four Instructional Weaknesses in Mathematics

The four instructional weaknesses to which connectionistic theory has contributed directly or indirectly are:

1. Our attention as teachers is directed away from the processes by which children learn, while we are over-concerned about the product of learning,
2. Our pace of instruction is too rapid while we fail to give learners the aids they need to forestall or surmount difficulty,
3. We provide the wrong kinds of practice to promote sound learning,
4. Our evaluation of error and our treatment of error are superficial.[4]

1. Process vs. product.—I have said that connectionistic theory leads us to neglect the processes by which children learn. This is so because, unlike the *product* of learning, the *process* of learning seems scarcely worthy of attention. The reasoning is somewhat like this: all learning is the formation of connections; or, the process of learning *is* the making of connections. Connections all being basically alike, the process is the same for all learning. Hence, we need only to make sure that the right connections are established; they will then necessarily lead to correct responses. We know what the correct responses are; we identify them for learners, along with the appropriate stimuli, and we provide practice until the desired connections between the two are formed. The process of learning, which is to say, connection-forming, takes care of itself. From all this we come to teach by telling children or showing children what to do and then by seeing that they do it.

We tell children that 2 and 5 make 7; we show them how to divide one fraction by another; we give them the rules governing signs in algebraic operations; we furnish them the facts of the Pythagorean Theorem. Practice follows to establish the

[3] Of all the exponents of connectionism Gates has argued most cogently in this vein. Arthur I. Gates, "Connectionism: Present Concepts and Interpretations." *The Psychology of Learning*, Chapter IV. Forty-first Yearbook of the National Society for the Study of Education, Part II. Bloomington, Ill.: The Public School Publishing Co., 1942.

[4] In the literature of connectionism I can find statements which contradict all these charges. That is to say, connectionists recognize, in theory, the evils which are listed above. But the habit of thinking of learning as connection-forming almost inevitably oversimplifies learning and affects teaching adversely. Perhaps no better example of this tendency is to be found than in the books *The Psychology of Arithmetic* and *The Psychology of Algebra*, both written by the originator of connectionism, Professor E. L. Thorndike.

connections. When our pupils demonstrate that they have the desired connections by producing the correct responses, the teaching job is done.

Now, more is the pity, some children try to learn mathematics according to this simple pattern. I shall have more to say about these children at a later point. It is pertinent here, however, to note the attitudes which such children develop toward mathematics. The correct answer is their sole consideration. Let them, by no matter what curious manipulation of symbols, arrive at an answer which agrees with that of teacher or textbook, and they feel that they have met all requirements of the situation. Change in the slightest degree the conditions in which the mathematics occurs, and they are helpless. Challenge an answer even when it is correct, and they have no way to prove it. To tell them that mathematics, whether it be arithmetic or algebra or geometry, is a system of logical relations is to speak in a foreign language.

But a large per cent of children do not learn mathematics in this simple, blind way, even when the teaching might seem to encourage such learning. Instead, they try somehow to put sense into what they learn. They may be told—and told time and again—that 2 and 5 make 7; but they forget it. When the forgetting becomes too embarrassing, they try something beside memorization. They turn 2 and 5 around to make 5 and 2, which for some reason they may know better; or they count one number onto the other as a base; or they see 2 and 5 as the same as 3 and 4, with a sum of 7.

You are probably all familiar with the girl reported by Stephenson, a girl who has many mathematical relatives in every community in which I have lived. This girl found verbal problems too much for her; or, they were too much for her until she devised some simple rules: when the problem contains several numbers, you add; when it contains two long numbers, you subtract; when the larger of two numbers exactly contains the smaller, you divide; otherwise, you multiply. This girl, like her many previously mentioned relatives, has been held up as a conspicuous example of stupidity. I cannot agree. I believe, instead, that she showed an extraordinary degree of originality and resourcefulness. As a matter of fact, barring computational errors, she would, by her procedures, get the correct answers for the majority of problems in texts for the lower and intermediate grades. And if she got the wrong answers? Well, so did the other children who had different "rules." It just happens that the processes used by this girl are not mathematical, and she was supposed to be working in the area of mathematics. But mathematics as such meant nothing to her; she had not learned the tricks of the trade; so, she invented some of her own.

What I have been doing, you will have recognized, is to illustrate negatively the importance of *process* in learning. Let us now approach the matter positively. Consider the example 42+27. There is not just one way to find the answer, as we sometimes naively assume. There are almost numberless ways:

a. One may count our 42 separate objects, lay them aside, count out 27 more similar objects, lay them aside in a separate group; and then find the total by counting all the objects by 1's, starting with 1.

b. One may use objects as in a, getting two groups of 42 and 27, and again count by 1's, but start with 43 or 28 instead of with 1.

c and d. One may use objects as in a and b, but count by larger units than 1's, as by 2's or 5's.

e. Still using objects, one may set up 42 as four groups of 10 objects each, with 2 over, and 27 as two groups of 10, with 7 over; he may then count the tens, getting 6, and the ones, getting 9, to yield the total of 69.

f to j. One may use any of the first five methods, substituting marks or pictorial symbols for actual objects.

k. One may count the 27 onto the 42 abstractly, or the 42 onto the 27.

l and m. One may copy the abstract numbers and get subtotals in the two columns by counting abstractly or with marks.

n. One may know the separate combinations and add directly: 2 and 7 are 9; 4 and 2 are 6; total, 69, without knowing anything about the composition of the numbers dealt with—a purely mechanical stunt.

o. One may proceed as in n, but be fully aware of the nature of numbers and of the process of addition.

p to N. One may do any one of the many things which this audience would report as their processes, such as: (1) direct and immediate apprehension of the total; (2) adding 20 to 42, and then adding the remaining 7; (3) adding 40 and 20, adding 2 and 7, and then combining; (4) adding from the left, with a preliminary glance to make sure that no carrying is involved; and so on, and so on.

All these procedures, and others not here catalogued, are entirely legitimate. All of them, except the last few, may be found in actual use in classrooms in which the procedure for adding such numbers as 42 and 27 is being taught. The teacher knows the *product* she is seeking to attain, namely, skill in adding two-place numbers without carrying. If she thinks of learning purely as the formation of connections, she is apt to oversimplify the learning task. She will tend to show her pupils how to add the digits in the two columns and where to place the partial sums; and then she will rely on practice to establish the needed connections.

I have tried to show that identification of the learning process with the formation of connections, however valid for ultimate psychological and neurological theory, is not useful to teachers. Teaching is the guidance of learning. We can guide learning most effectively when we know what the learners assigned to us really do in the face of their learning tasks. In a word, we as teachers can be helpful in guidance to the degree to which we know our pupils' processes. I do not mean that the product of those processes is no concern of ours; but I do mean that processes are of at least equal importance with products. The teacher who knows the product which is to be finally achieved, but who also knows how to discover, evaluate, and direct the processes of her pupils as they approach this goal—that teacher is probably a good teacher. Moreover, thinking of learning as the formation of connections would not make her a better teacher.

2. Over-rapid instruction.—So much for the first objection to the connectionistic view of learning: it takes us as teachers away from our main stock in trade, namely, the processes by which children learn. The first objection is closely related to the second: it tends to make us hurry unduly the pace of instruction and it discourages us from supplying to children temporary aids and procedures which they need for sound learning. In a word, we are led to think that children can complete their learning at a single jump.

Without reciting them again, let me recall the list of processes by which one may find the sum for 42 and 27. These processes were described in the earlier place only to establish the existence of various possible processes. Let me cite them again here for another purpose. The processes were arranged by plan, in a roughly ascending order of complexity, maturity, and abstractness. It is surely obvious that the child who counts by 1's 42 objects, then 27, and finally the total of 69 is acting more simply and concretely and less maturely than does the adult who apprehends at a glance the sum of the two abstract numbers. And the other processes listed can be posted at intermediate points in the scale of maturity and abstractness. Any given child may be at any point in this scale. Furthermore, his degree of mastery of his process, whatever it is, may vary from inexpertness to strik-

ing proficiency. Indeed, a child whose procedure (counting by 1's, for example) is very low in the scale of maturity may outscore in rate and accuracy another child whose procedure is at a higher point in the scale. Failure to note this fact is one of the penalties we pay for neglecting process in favor of product only, and it constitutes a large source of error in the evaluation of learning.

We have come to accept the typical curves of learning as picturing all that goes on in learning. The accuracy curve, for example, mounts rapidly at first and then slower and slower. The implication is that the child is getting the desired connection established just about as is portrayed in the curve. But when we examine into the behavior of the child, we find the situation to be much more complex than this. He may practice a given procedure for a time and then desert it for another, and this for another, and that for still another. A more valid picture of his learning, this time plotted in terms of process, would look something like a series of steps, each successive one somewhat higher in the scale of maturity than the preceding one. In a word, the learner progresses by traversing a series of stages in thinking. Each stage serves its purpose for a time, but is superseded by a more advanced stage. As each stage is abandoned for the next, the earlier stage is not forgotten or gone—its pattern is not eradicated from the nervous system. Instead, the older procedure is overlaid by another, and the old neural pattern remains for use if for any reason the more recently acquired procedure does not function smoothly.

This description of learning, while it may be consistent with connectionism, is certainly not suggested by connectionism. On the contrary, the connectionistic view of learning leads us to give the child at the outset the form of response which we want him ultimately to have. And we are inclined to do this quite without regard to his attained stage of thinking when we present the new learning task. At best, the result is pseudo-learning, memorization, and superficial, empty verbalization.

Let me illustrate what I mean. I once asked a third-grade teacher to send me for interviews the three poorest arithmetic pupils in her class. George was one of the three. He was described as an almost certain failure. After some preliminary exercises, I put before George the three digits 8, 7, and 9 arranged for column addition, and asked him to find the sum. Listen to him: "8 and 7 are 15, and 9 are ... I don't know." Then, in the reverse direction: "9 and 7 are 16, and 8 are ... I don't know." After a pause George looked at me and asked, "May I count?" I told George he might, and promptly he mumbled, "8 and 7 are 15, 16, 17, ... 24." When I asked him to add upward, he produced the correct answer with equal quickness by counting the 8 to the sum of 9 and 7. Obviously, George did not understand bridging of the decades, save as this could be done by counting.

But why did George ask if he might count? Clearly because his teacher had told him that he must not count, with what penalties I do not know. George obeyed her, and the reward for his obedience was failure. Parenthetically may I say that one of the three *best* pupils sent from the class for interviews counted not only to bridge, but counted to find the sum of each pair of digits. This last child, singled out for praise, was really at a lower stage of thinking than was George; but the teacher did not know it. She had asked these children to perform at a level or stage of thinking of which they were then incapable, and they had reacted according to their natures.

To return to George and to repeat what I have already said—this lad was at a very immature level with respect to bridging: he had to count if he was to bridge at all. I said to George, "George, how many are 8 and 7?" He said at once, "15." I asked, "How many more does it take to make 20?" He said promptly, "5"; and then after an instant his face lit up and

he said, "And 4 more are 24!" Then, "Will it work like that every time?" I asked him to add the same numbers upward, and he said, "9 and 7 are 16; . . . and 4 are 20, and 4 are 24! Can I do that with all of them?" I wrote several examples which called for bridging 20, and he solved them quickly, understandingly, and triumphantly by his new method.

What had I done? Well, I had helped George to the next higher stage in thinking, a stage for which he was ready, but a stage which he had not discovered for himself and which he was unlikely to find, so long as instruction consisted only in telling what he must not do.

Should George have been left at the stage to which I had helped him, always to bridge 20 or some higher decade by splitting a number? No; he should have been led next to understand the principle of adding by endings, and still later to think immediately and directly of the total of the last partial sum and the last digit. Sound learning required that he traverse these intermediate steps; he was getting nowhere by his own devices. Unfortunately circumstances prevented my helping him discover the next steps, and I fear that his teacher may not have provided the needed assistance either.

I have spent all this time on George because his case illustrates so clearly the dangers of too rapid a pace of instruction. It illustrates too the need for temporary or intermediate processes, methods, and devices which are now commonly kept from children to their detriment. These temporary aids have a bad reputation in education. I am sure you have often heard the dictum: "Never form a habit which must later be broken." The addition of the qualifying phrase "Other things being equal," whatever its intent, does little to soften the ban against their use. Indeed, so fully are school officials and teachers persuaded of the evils of these aids that they are to be found in few textbooks, and there in insufficient amount. I do not mean however to imply that these aids are totally unknown to the classroom. Good teachers use them—sometimes openly, perhaps more often when the supervisor or principal is not likely to appear.

So long as we think of learning as a simple, straight-line development, the warning against temporary aids makes sense. These aids seem to contribute little; they increase the number of things to be learned; they tend to be retained after they have long outlived their usefulness (if any). But we must cease to think of learning, and certainly of learning in mathematics, in this manner. We do not seek merely to develop a few mechanical skills always to be used precisely as they were learned. The purpose of mathematics, whether in the elementary school or in the high school, goes far beyond the establishment of mechanical skills. The ideas, principles, generalizations, and relationships which are taught, *as well as the skills*, are intended for purposes outside themselves and for use in situations quite unlike those in which they are learned. We teach quantitative mathematics, for example, as a system of thinking by which to manage and control number and quantity, not alone as presented in textbook problems or as presented in the classroom, but however and wherever and whenever presented. In a word, we strive to teach understandings. When the goal of understanding is accepted, the function of temporary aids is seen in its correct perspective. Such aids contribute meanings when meanings are needed; and the more meanings, the deeper the understanding, and the greater the chances of successful transfer to new and unfamiliar situations.

In making a case for temporary aids I may have failed to indicate all that should be included in this category. My illustrations have all been what may be called "lower-order thought procedures." But there are others as well. Here belong also the techniques and materials commonly embraced by the term "sensory aids"—drawings, pictures, maps, films, diagrams, slides, solid articles intended primarily for

classroom use, the geometric forms of architecture, and so on.

The argument for the abundant use of such aids is precisely the same as that for lower-order thought procedures. Learning is progressive in character. The abstractions of mathematics are not to be attained all at once, by some coordinated effort of mind and will. Instead, we must start with the child wherever he is, at the foot of the ladder, or at some point higher up. Well chosen sensory aids reveal the nature of the final abstractions in a way which makes sense to the child. If he can work out the new relationships in a concrete way and can himself test their validity in an objective setting, he has faith and confidence at the start; and he is the readier to learn with understanding the more abstract representations of mathematics. Sensory aids, like many so-called crutches, are then not only admissible under the conception of learning which I am outlining: they are obligatory.

On the other hand, the connectionistic view of learning does not predispose the teacher to employ temporary aids to the extent to which he should employ them. Instead, for reasons which I have already mentioned, it leads him too quickly to abstract practice or drill. And this is the third objection I have listed for consideration.

3. Faulty practice.—It would be false to accuse connectionists of spreading the gospel that "Practice makes perfect."[5] Nevertheless, the teacher who accepts their view of learning comes easily to reply upon drill as his exclusive or major teaching procedure. Listen again to the litany: Identify for the learner the situation to which he is to react, and the response he is to make to the situation; have him make the connection under satisfying conditions; have him exercise the connection until it is firmly established. Does not this *sound* like a exhortation to drill?

The apparently innocuous statement, "Practice makes perfect," is full of dynamite because it conceals important issues. Practice *does* make perfect in one sense of the words "practice" and "perfect"; but it makes for superficial learning in another sense of these same words. I can make my point by using a crude illustration.

Suppose that I, an amateur at golf, dub my drive: the ball barely trickles off the tee. What do I do? Practice? If so, in what sense? Well, if I practice in the sense of *repeating* my act, I grasp the club the same way, stand the same way, and swing the same way. With what result? I shall certainly get the same result—with this difference: I shall have become a bit more proficient in my poor drive. That it to say, there will be a little more economy and ease in my movements; I shall be more certain of what will happen; I may swing a little more quickly and precisely. Now, suppose I continue my repetitive practice over a period of time. Eventually, I shall become the most efficient poor golfer on the course, for repetitive practice will have brought its consequences.

But, you say, no golfer would do anything so silly as this; he would know what would happen. At first his practice would not be repetitive, but varied. That is to say, he would try to *avoid* the first combination of movements which produced the poor drive. He would stand differently, hold the club differently, swing differently, and so on. Only when he had arrived at the best possible combination of movements, as judged by the kind of drive it produced, would he start repetitive practice. And he would repeat purposely, because he would know that repetition could now give him the efficiency he desired.

But you are teachers of mathematics. What does this little excursion into golf have to do with the teaching of mathematics? At the risk of being dogmatic, I should say a good deal. As a matter of fact, we learn motor skills (the drive in golf) about as we learn abstractions (mathematics). It is only because we cannot directly observe the activities in idea-

[5] Some connectionistic accounts of learning contain explicit warnings against over-reliance upon repetitive practice.

tional learning that we come to think them different for motor learning. However, the appearance of difference vanishes when we adopt appropriate techniques for observation. Then we see the essential sameness in the case of motor skills and of abstractions. In both instances learning is characterized by the organization of behavior at successively higher levels. It is because of this essential sameness that the illustration from golf may prove helpful in showing the real function of practice in learning mathematics.

We are now ready to formulate two statements which should supplant the familiar "Practice makes perfect." (a) "*Varied* practice leads to the discovery of the right combination of movements and ideas." (b) "*Repetitive* practice produces efficiency, but at whatever level of performance the learner has attained." The proviso in the second statement is highly important: if repetitive practice is introduced prematurely, the learner is "frozen" at his level of performance. He steadily becomes more proficient at an undesirably low level of maturity. Repetitive practice cannot move the learner to a higher or more mature level. If, under conditions of drill or repetitive practice, the learner does actually move on to a higher level, the credit does not belong to the drill to which he has been subjected. An examination into his behavior will reveal that he has deserted the prescribed repetitive practice and has struck out for himself into varied practice.

It follows from what I have just said that in the end we defeat our own purposes by introducing repetitive practice too soon. In the early stages of mathematical learning we need to institute activities which will enable the learner to explore understandingly the new area which he is entering. The learner is not exploring this area when he does nothing but repeat what he has been told or solve problems in the way in which he has been shown. Continued practice of this kind can yield nothing more than superficial learning: his efficiency may mislead us into thinking that he has a more thorough grasp than he actually has.

It also follows from what I have said that there is plenty of room and a great deal of need for repetitive practice in mathematics. Extremists in education who have reacted quite properly against premature drill do not correct the evil when they say that drill has no place at all in modern teaching technique. Such individuals would certainly engage in repetitive practice in golf; they would not for a minute believe that the limited insight of one lucky drive would give them command of the stroke. Yet, they would abolish drill in ideational learning. But drill is an inappropriate teaching procedure only when it is called upon to do what it cannot do. It is entirely appropriate when the goal is efficiency.

4. Remedial teaching.—We come now to the fourth objection to the prevalent conception or misconception of learning: uncritical acceptance of connectionism is likely to lead to the misinterpretation of error and to the use of inappropriate remedial measures.

When a child makes a mistake, it is easy, but misleading, to say that he has made the wrong connection or that he has failed to make the correct one. And it is easy, but wrong, to assume that that mistake can be remedied merely by showing the child the correct response and having him practice until a connection is formed. This analysis of error and the proposed remedy might suffice if the learner were dealing with nonsense syllables or with puzzles which he could solve only by accident.[6] If

[6] As is well known, the connectionist *as a connectionist* (that is, when he is concerned primarily with theory) has experimented chiefly with just such experimental problems. Even in such learning situations as he sets for the learner the connectionist would, if he but attended to what the learner does (his process), discover that learning is much more complicated than it appears in his account in terms of rate and accuracy. He would discover, even in the case of nonsense syllables and of mazes, that the learner approaches the final goal of efficiency through a process of reorganization at succes-

the learner fails to recall one of the syllables or if he makes the wrong turn in a maze, we give him the correct syllable, or we tell him his error in the maze or show him the correct turning. Remedial instruction in such learning assignments seems to be simple indeed: tell or show, and practice.

But the classroom should present exceedingly few learning tasks of these kinds, particularly in mathematics. Of course the learner must memorize arbitrarily predetermined characters and symbols, and in this case remedial instruction consists primarily in telling or showing and then in practicing. But, typically, learning tasks in mathematics are far more complex since they involve meanings and understandings.[7] I believe you will agree that most errors in mathematics are the result, not of imperfectly learned symbols, but of incomplete understandings, of inappropriate thought processes, and of faulty procedures.

Consistently throughout this paper I have stressed the progressive character of learning: the learner moves from level to level in thought processes, each successive level being more mature, more abstract, more adult-like than the preceding. Except in the case of imperfect mastery—and drill is then the remedial measure—except in such cases errors come from failure to traverse the stages and levels of thinking in an orderly fashion. Called upon to perform at a level higher than any he has yet attained and given no guidance to reach the higher level, the child has but three courses of action open to him. (a) He can refuse to learn. Refusal may take several different forms. One form is, "I won't." Under ordinary conditions of schooling this form is not common. Another form is, "I can't"; a third is, "I don't want to," or "I don't care." The result of refusal, by whatever form, is indifference toward mathematics or dislike of it, which may be accompanied by widespread feelings of frustration.

If the child does not refuse to learn, he may adopt a second method of extricating himself from his predicament. (b) He may do his best to perform as he is asked to perform. This method is probably the commonest, and it reflects the effects of years of training in docility. Of course the child cannot actually do what he is supposed to do, but by blindly following rules and by profiting from model solutions he may *seem* for a time to be successful. His answers are correct, and he gets them with reasonable promptness. This evidence of learning, based upon the criteria of rate and accuracy, is spurious. The skill which is developed will not be useful except in the situations in which it is learned, and after a short while even this degree of skill is gone.

(c) The child's third method of surmounting his difficulty is to fool his teacher, to continue actually to perform at his attained (but not at the expected) level, but to conceal this fact. This method is successful when the child develops enough proficiency with his lower-order procedure to equal the performance of other children who are higher in the scale of maturity. And it is not impossible to attain this degree of proficiency. Many children do, and the fact is not discovered until later, perhaps a year or two, when with the growing complexity of learning tasks the low-order procedures are no longer effective.

It should be clear that no one remedial technique will be successful with these different types of disability. The child whose attitude toward mathematics has been ruined needs to have that attitude corrected. The working of masses of unenlightened and unenlightening examples

sively higher levels. Gates has recognized this fact to a far greater extent than have most connectionists. See: Arthur I. Gates, *Psychology for Students of Education*, Revised Edition. New York: The Macmillan Co., 1930. Chapter XI.

[7] The connectionist of course agrees, but he has what to him is an explanation of the complexity. The learner is merely forming many connections at the same time, concomitantly and in succession. I can agree that this may possibly be so. At the same time I find little in this view which provides guidance to the teacher.

and problems will not reach the source of difficulty. If the undesirable attitude arose because of inability to understand and of a consequent series of failures, the child's attitude will improve only when he understands and when he has had ample experiences of a successful kind.

Similar treatment may be necessary for the child who has developed spurious skills, only to lose them. More drill will serve only to revive for a time the skills which never were worth the trouble to cultivate. It is for this reason that so-called refresher courses in mathematics in the high school must frequently fail of their object. What most high-school students need is not more of the kind of instruction which produced their deficiencies, but a type of instruction of which they have not had enough. If the giving of this instruction means that the usual systematic mathematics courses in the high school must be sacrificed for many children, what of it? With the appallingly weak foundations many high-school students have in earlier phases of mathematics, they can hardly profit from the more advanced phases to which they have been traditionally exposed.

I have considerable sympathy for the child who has adopted the third way out of difficulty, that of remaining at a low level of performance and of developing proficiency therewith. After all, whatever he does, and however far he may be from the desired level of performance, he knows what he is about. In this connection let me recall the case of George who had trouble with bridging the decades in addition. The way to secure a higher level of thought process in such cases is certainly not to deny the child knowledge about that higher process, at the same time forbidding him the only process he has. Nor is it any more effective to assign large bodies of drill, for, in the case of less honorable children than George, the drill examples are but so many more invitations to increase facility of performance at a low level. Here as elsewhere, remedial measures must accord with some particular kind of deficiency.

I cannot quit this matter of remedial instruction without commenting upon certain practices which I have witnessed. I am both amused and irritated by the behavior of some college professors of mathematics when their weaker students come to them for help. Observe them, or observe a typical pair. The professor greets the confused student pleasantly; he sits down with him at a table, turns to the section of the text which is causing trouble, and neatly copies an example on a nice, clean sheet of slick white paper. Then they go to work. But before we come to the "work," let us note that the professor copies the example with a *pen*, and he does all his work with a *pen*. Now perhaps memories of my own difficulties in college mathematics make me hypersensitive, but I resent that pen! I claim that the pen adds insult to injury. Consider the gulf its use establishes between student and professor. The student does his mathematical work with a pencil which is equipped with a large, competent eraser—and he uses that eraser frequently and vigorously. But the professor! he is so sure of himself that he can use a pen, thereby making a record which cannot easily be altered—only of course the professor knows that he won't have to alter his record. So does the student, and this knowledge may shake still more his already wavering self-confidence.

But I have admitted that my prejudice against the pen may be purely personal. So, let us get back to the work. What happens? The professor writes out each step in the solution calmly and certainly, meanwhile accompanying himself with a monologue. I cannot say that he is carrying on a conversation because the student contributes no verbal comment. Indeed, it is to be questioned whether he contributes anything at all, including understanding of what is going on. At the conclusion of the exhibition the professor settles back, well pleased with himself, and says mildly, "Well, there it is. Do you see how to do

it?" Courtesy alone would require the student to say that he did see it, even if the stupefaction which magic produces did not render him incapable of more than nodding his head weakly or saying merely, "Yes."

Whatever name we give to this seance, we cannot call it remedial instruction. It might be a good idea to deny the professor all use of pencil and paper—and certainly of pen and paper! The student has come for *help*, not for a demonstration of the professor's skill in mathematics. If the professor were unable to write out his own processes, he might give the student the kind of help he has come for. The student, not the professor, should do the work. He should go as far as he can on his own; when he can go no further, he should be questioned and guided through questions to locate his difficulty and to analyze its nature. Through continued questioning he should be led to suggest possible next steps and then to evaluate these steps himself. But at all stages the student should be required to make use of his own knowledge (to the extent that he has any) and he should be allowed to identify his deficiencies himself and to feel that he is making progress by his own efforts. Remedial instruction of this kind is worthy of the name, and the results justify the time and energy that must be expended to secure them.

So much for the four instructional weaknesses which I listed for discussion, weaknesses which, if they cannot be attributed to the connectionistic view of learning, have certainly not been dispelled by the general acceptance of this view in American education. At the outset I promised that my comments would be constructive as well as critical. I think I have kept my promise. For a conception of learning which may be helpful so long as we deal with the most uncomplicated types of learning I have offered a substitute which may be more helpful for the kinds of learning which are involved in mathematics.

This latter conception stresses the notion of progressive reorganization. It emphasizes the essential continuity of learning. It points out that the learning of relationships and the development of meanings take time which is filled with suitable activities. It defines the kinds of practice which are appropriate at different stages in learning. It guarantees that children will not be hurried toward empty verbalizations but will be directed toward useful abstractions. In a word, it is a conception which provides insights into the course of learning which children must pursue if they are to attain the approved objectives of mathematics. I commend this conception to you teachers who are charged with the responsibility of guiding children to a meaningful and intelligent grasp of mathematics.

The Old Time Religion*

By Evan A. Nason
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In order to furnish a background for what I have to say this morning, may I ask you to go back with me to the period of American History sometimes called the "Roaring Twenties." This decade from 1920 to 1929 was a fantastic period in our history. It was a period of fads and fancies; a time of revolution in dress and manners; a period of prosperity, easy money and unconventional thinking and living. This was the decade of mahjong and crossword puzzles; of Coué and his magic formula "Day by day in every way I am getting better and better." Vitamins had not yet been discovered but prohibition was in effect and deficiencies in vitamins *A*, *B*, *C* and *D* were more than compensated by liberal doses of wood alcohol.

But first and foremost this was a decade of tremendous business expansion in which the automobile took the lead. "A chicken in every pot and two cars in every garage" was the slogan of the automobile salesman. (Evidently the chicken was thrown in as a sort of bonus.) Salaries, except for teachers, were progressively increased, and it was the exceptional family that depended solely upon income from salaries. A few hundred dollars invested in Radio Corporation of America might yield more in a few days than the head of the house could earn in a year. Executives of "big business" corporations became national heroes and boys, who in earlier years might have planned to become doctors, lawyers or even teachers, now looked forward to following in the footsteps of these favored leaders. Prosperity and the steadily rising stock-market insured the highest material standard of living the world had every known; but at the same time there was a corresponding decrease in the influence of the home and in the influence of the church.

It was inevitable during such a period, when people were concerned principally with material comforts and pleasures and questioning the value of everything else, that traditional forms of education should be assailed. The subject of mathematics seemed to bear the brunt of this attack.

The frontal assault was led by the taxpayers who supported the public school system. The "Bread-and-butter" variety of education was their principal concern at this particular time, and a large part of mathematics as then taught seemed to the tax-payers lacking in practical values for the majority of boys who were destined for the field of business.

Flank attacks were carried on simultaneously by many of the leaders of educational thought in this country. These educators held many and varied theories concerning the place of mathematics in the school curriculum. Many realized that the school curriculum was too narrow and that new interests should be supplied, time for such new subjects being obtained by decreasing the content of the regular school subjects.

A great many educators felt that arithmetic was too difficult for the average child in the grades and that everything of a difficult nature should be omitted retaining only those parts of arithmetic which could be shown to be of social significance. They argued that failures early in life retarded the healthy development of a child's personality and individuality.

And supporting the frontal and flank attacks the psychologists unlimbered the heavy artillery massed in the rear and laid down a terrific barrage. The psychologists attacked the theory of transfer of training and conducted experiments which proved

* Paper read at a meeting of The Association of Teachers of Mathematics in New England held in Boston University—at Boston, Massachusetts on December 4, 1943.

to the satisfaction of many of the investigators the absence of transfer of any kind. Thus the cornerstone upon which the inclusion of mathematics in the educational program had been based seemed to have been shattered by the big guns of the psychologists.

Attacked thus in the center by the taxpayers, on the flanks by the progressive educators, and with the shells of the psychologists bursting overhead, the unhappy teachers of mathematics, as the German Ministry of Propaganda so quaintly puts it, "adopted an elastic defense"; they endeavored to "successfully disengage themselves from the enemy"; and they "energetically moved in a new direction."

By this time the decade of the "Gay Twenties" was over. The great bull market of the Republican party had collapsed carrying the Republican party along with it. The two cars had been repossessed by the Finance Corporation and the owner of the garage was frantically searching for a purchaser for the garage itself. The aroma of the chicken roasting in the oven became only a pleasant memory. Prosperity was still around the ever-elusive corner. Salaries were decreased, and this time the teachers were not forgotten.

Other factors resulting from widespread unemployment and a marked decrease in the national income aggravated the decline of substantial work in mathematics initiated in the preceding decade. During the latter part of the thirties many independent schools had been forced to close because of lack of patronage; others were operated with an insufficient staff; and it was not an uncommon occurrence for whole city school systems to declare vacations of several months' duration because of lack of money for operating expenses.

Now I know that you are thinking that if the position of mathematics was in reality as desperate as I have implied in this roughly sketched-in background, how did mathematics survive at all? Why did not the breed of mathematics teachers become extinct? How indeed did this country continue to carry on normal peacetime pursuits?

If you will think back just three years ago you will all, I am sure, have to admit that the condition of this country was in truth precarious. Germany was overrunning Europe and offering a challenge to this country as well; Japan was threatening from the Pacific; and at home the Democrats had just been returned to power for another four years.

But I believe that there are a few things upon which Democrats and Republicans can both agree. In the twenty years before the opening of World War II there was a marked tendency, becoming progressively more pronounced, towards "de-emphasis" of mathematics as a school subject. The influence of mathematics decreased as the school program was broadened to include many courses in social studies and other subjects.

Unfortunately the laudable effort to broaden the program was accompanied by a loss of depth. It was my personal experience that the search for a sound foundation upon which to build what I hoped was to be a substantial course in elementary algebra led ever deeper and deeper. On one of these diving expeditions I asked one of a new group of pupils, "What is the sum of $\frac{1}{3}$ and $\frac{1}{2}$?" After several seconds' thought the boy replied, "I am not sure if it is one-fifth or two-fifths." Although I considered both suggestions incorrect, the boy's thoughtful, questioning attitude was distinctly encouraging and led me to believe that here was a searcher for the eternal verities.

But of even more fundamental importance than general weakness in fractions was the lack of a real concept of the number system as a basis for understanding and organizing later material. What might be called a "number sense" was lacking. These observations, based as they are upon results obtained in school systems of three cities, *not in New England*, within a radius of twenty miles, are of no significance if the situation was a local one only.

Evidence gradually developed, however, to create the suspicion that this was not merely a local condition. College professors from all over the country complained of difficulties encountered in teaching advanced mathematics to students deficient in the fundamentals of algebra and geometry. Secondary school teachers pointed out the difficulties involved in teaching algebra and geometry effectively to students unfamiliar with basic concepts of arithmetic. This was a variation of the "old army game" with the first grade teacher in the role of the unfortunate buck private.

And this suspicion that all was not well in the realm of mathematics became a certainty when results of Army and Navy tests conducted on a nation-wide basis were published. May I quote from the most recent report on "Essential Mathematics for Minimum Army Needs."[1]

> The typical inductee does not have the training in mathematics which he needs. An accumulating, if distressing, body of evidence supports this statement. When only one inductee out of four can select the correct answer from four suggested answers for 5 is 20% of what number; when only one in three can select the correct answer for $7-5\frac{3}{4}$; and when only one in four can select the correct answer for .32 divided by .64; under these conditions it is clear that the inductee is ill prepared to cope with quantitative situations he will encounter in his basic training in the Army.

I have dwelt at some length on the vicissitudes of mathematics, arithmetic in particular, during the twenty year period before the present war, because, if mathematics is to be taught effectively and is to fill the important place in education it should fill by virtue of its fundamental necessity in this modern scientific age, there must be developed in this country a consistent philosophy of teaching mathematics. No progress can be made if there is one philosophy for elementary schools, another for secondary schools and still a third for the colleges and universities.

Looking upon the past two decades, it becomes evident that it was a period in which educators and teachers were groping and struggling to formulate just such a philosophy. It is not surprising in a country the size of the United States, with its heterogeneous population, with a freedom of thinking not influenced by one central and official propaganda ministry, with such diversified and divergent views of the fundamental aims of education, that the evolution of a philosophy of teaching mathematics which could be generally accepted was accompanied by a great deal of confusion—truly an understatement of actual conditions.

But toward the end of the thirties, out of the amorphous state of the preceding years, a more definite and consistent philosophy of teaching mathematics began to crystallize. Some theories, theoretically logical and plausible, had not withstood the test of practicability when subjected over a period of years to actual teaching conditions. On the other hand the insistence of the progressives that the barrier between the theoretical side of mathematics and the practical side must be broken down led to the elimination of obsolete material and to the adoption of a more sane and vital viewpoint of the subject in its entirety. Determined efforts had meanwhile been made to add to the meager knowledge of the psychology of human learning and to make teaching methods conform to the most recent theories of the psychologists as to the mental processes involved in learning. This crystallization of a more consistent philosophy of teaching was, of course, accelerated by the advent of war which focused the attention of the whole country upon the importance of mathematics in time of war and its many applications in time of peace.

I would like to enumerate certain facts and principles, the validity of which, it seems to me at least, has been demonstrated by the experimentations of the last two decades. A summary of principles of importance in the teaching of any subject is naturally influenced by the personal be-

[1] THE MATHEMATICS TEACHER, October 1943, page 244.

liefs and experiences of the writer. This case is no exception. It is not improbable that many of you may not agree with all of the conclusions here drawn. Disagreement will mean at least consideration and some benefit may result from such consideration.

1. There are very real and definite reasons for the inclusion of mathematics in a school curriculum wholly apart from the practical values gained from its study.

Not being a psychologist, I certainly do not intend to discuss the question of transfer of training over which psychologists have argued for the last fifty years. The principal reason for the contradictory views held seems to be the absence of a precise definition as to what is meant by the expression. Possibly a more thorough training in mathematics would have obviated this difficulty.

There are, however, certain ways of thinking, certain processes of thought, certain methods of procedure which are important in all fields of human endeavor. For the development of a great many of these processes of thought mathematics is peculiarly suited. The habit of logical and neat arrangement of work; the necessity for care and paying attention to detail; the necessity for precise reading and accurate statements; the accurate definition of terms; the insistence upon reasoning rather than intuition; the reasoning by *reductio ad absurdum*—these are a few of the values obtained from the study of the subject of mathematics. Surely these processes once learned are not forgotten and do carry over into other fields. Incidentally the latest researches in the psychology of learning support the presence of transfer as suggested here.

2. The elimination of material merely on the grounds that it may present difficulties is pedagogically unsound and contrary to the desires of the pupils themselves.

Unequivocally and without reservation do I take exception to the premise that has been one of the principal causes for the decline in substantial work in mathematics during the past fifteen years—namely that the inclusion of difficult material is harmful in the development of personality and results in the acquiring of a dislike for school and education in general. To expose a child or an adolescent to a program from which all difficult material has been eliminated is to accustom him to experiences wholly unlike those of later life. Furthermore it has not been my experience that boys shun or dread the difficult parts of mathematics. I have had boys just beginning the study of geometry spend weeks (not at my suggestion) investigating the possibilities of trisecting an angle. Occasionally I have assigned lessons consisting of two parts—the regular lesson followed by an optional question more difficult than the regular part. The regular lesson might be neglected but never the optional question, and I have frequently been amazed by what a slow pupil could accomplish on a question of this kind. There is probably more danger that interest be lost by not supplying material that offers a real challenge to a boy than there is in the inclusion of too difficult material, provided, of course, that the teacher has made proper preparation for a concept he knows will offer difficulties.

That the youth of today does not shirk difficult assignments has been conclusively demonstrated by achievements of young men now in service all over the world. These young men in the space of a few months have had to become familiar with the idiosyncrasies of the internal conbustion engine; they have had to study communications, meteorology, advanced navigation and the multitude of things a modern pilot, navigator or bombardier must know. They have done all of these things and have become the world's best-trained pilots in spite of deficiencies in elementary mathematics, caused to a large extent by this desire to shield them from difficult tasks, a theory to which the young men themselves never subscribed.

3. If difficult work is to be included in

courses, great care should be taken in the teaching of such concepts to insure real insight and understanding by the pupil.

Recently the theory of "meaningful teaching" has been much emphasized. The emphasis in this theory is on thorough and careful teaching so that the pupil has a real understanding of the problem and does not depend upon memory alone. With this point of view there can be no disagreement, I am sure. Without real understanding the difficult, which I do not view with alarm, becomes the impossible for which there can be no justification.

4. Drill work is necessary for real understanding of concepts and ability to put into practice basic ideas.

I use the expression "drill work" deliberately because there has been so much objection in recent years to anything that suggests drill. By drill work, however, I most certainly do not mean the mechanical repetition of certain facts or the mechanical development of skills based upon concepts not clearly understood. Rather by drill work do I mean the extension and amplification of a basic concept which has first been taught meaningfully. In this kind of work, a large part of it oral, the teacher leads the class to see how the new idea is actually used; by many and varied applications the new concept is made a part of the mental equipment of the pupil, ready for use when opportunities arise. Such work, although definitely drill work, is by no means dull and uninteresting. In fact some of the most interesting recitations develop from this type of practice work.

5. Applications of mathematics to practical problems of war or peace should follow study of basic principles.

Once the fundamental principle is understood the applications follow logically. It seems to me unsound pedagogy to teach applications of mathematical principles before the underlying concept is thoroughly understood. Certain skills can be developed by mechanical repetition without understanding. But if the conditions are changed, the student is lost unless the basic principle upon which the skill depends is understood. The applications should be used as practice work to add meaning and insight to the basic concept.

6. Courses in mathematics should be elastic enough in content to permit variation of material covered.

By elasticity of content I have in mind variations within different classes in the same school more than variations between schools in different sections of the country whose programs are naturally influenced by local needs, goals of the majority of the pupils, and many other factors. No two classes react in the same way to the same material. There seems to be no sure way of predicting the reaction of a class to any specific concept although the achievement over a period of months or a year may be fairly accurately determined in advance. I am sure that all teachers have had the experience of a good class having unaccountable difficulty with some concept easily understood by most classes. On the other hand I recall distinctly a class, which was definitely slow, doing exceptionally high grade work in the theory of exponents—the more difficult the problems, the better the class enjoyed them.

If this is true it would seem inadvisable to insist that all sections follow rigid assignments laid down for some time in advance. All branches of secondary school mathematics are well fitted for a more elastic treatment. In algebra certain cases of factoring, the simplification of complex fractions, difficult work in the *Theory of Exponents*, unusual types of solution of simultaneous quadratics, problems of a very difficult nature—all of these can be included or omitted depending upon the ability of the class. In geometry a great many of the less important theorems and the applications based upon them can be omitted without losing fundamental values or destroying the continuity of the course. The same thing is true to a marked extent in trigonometry. The important consideration is not how much to teach, but

how well the material covered is taught.

And finally; there must be a definite and continued effort to merge the different branches of mathematics into one well-integrated whole.

The walls that frequently exist between algebra and geometry or between geometry and trigonometry must be broken down. If mathematics is to be used effectively the student must have a firm grasp of the entire field. It is not sufficient to understand the different branches only. They must be welded into one strong whole.

Now perhaps you are thinking, "There is nothing new, nothing startling in all of this. It sounds pretty old-fashioned to us." I trust that I have made it clear that the philosophy of teaching here advocated is far removed from a philosophy sometimes associated with "old-time" methods, which limited the teaching of mathematics to the theoretical side and encouraged the development of certain skills by means of constant repetition, often with too little concern for real insight into the principles upon which the skills were based. If by "old-fashioned" is meant the belief that only by careful, thorough, and substantial work carried on over a period of many years, can there be developed a real power of analysis and the ability to use mathematics freely in practical situations—with such an interpretation of the term "old-fashioned" I am heartily in accord.

And, unless I have interpreted incorrectly the most modern teaching trends, these principles just enumerated are in full accord with recent developments and with present day requirements.

The immediate requirements of the day are, of course, those connected with our war effort. When, two years ago, this country found itself in war against the Axis, the educational system of the United States threw itself without reservation into support of the war effort. The only question that was asked was "What can the schools do to aid most effectively in the present emergency?"

Officials of the Army, Navy, and Marine Corps responsible for the program in mathematics for the drafted men as well as those in charge of specialized training, early made it clear that they did not recommend fundamental changes in the usual program of *Mathematics in the Secondary Schools*. Their concern was for men of sound mathematical training who could apply that training to the needs of the particular service in which the men were placed. It was recognized that the essential function of the secondary school was to provide a solid foundation upon which later developments could be built.

That this point of view has not changed during the past two years is evidenced by the report on "Essential Mathematics for Minimum Army Needs" referred to previously. The major portion of this report has to do with specific applications of mathematical principles to the needs of the average inductee. It is hoped and expected that teachers of mathematics will study this report carefully and see to it that during the present emergency these applications of a practical nature are covered in the courses they are teaching. However, the report does not recommend confining the teaching of mathematics to applications. On the other hand it emphasizes the necessity of teaching fundamental principles thoroughly before teaching applications. To quote again directly from this report:

> One does not have to become theoretical when one discusses meanings; meanings are really practical. It has long been pointed out that skills can not be used intelligently if they are not learned intelligently . . . . If this report seems to over-stress meanings and understandings, it is only because these mathematical objectives have been too long under-stressed.[2]

I have at home two letters, one from teacher of Navy recruits and the other from an Army officer, a graduate of Massachusetts Institute of Technology, now engaged in specialized training. One says "It is always bad policy to teach applications without first teaching the basic principle

[2] *Ibid.*

of any technical subject." The other points out that in the technical branches of the Army even after one and one-half years of college engineering, the student is called upon to take such subjects as Metallurgy, Mechanics, Machine Tool Processing, Descriptive Geometry, Advanced Physics, Casting Processes, Strength and Materials, and Kinematics. He observes, "it is obvious that a boy can not successfully tackle these subjects unless he has a fine basic knowledge of mathematics in elementary and secondary schools."

The conclusion that war-time needs do not require revolutionary changes in the program of mathematics in the secondary schools is inescapable. But it is also clear that the work in the secondary schools should be more thoroughly done and should be of a more substantial character than it has been in the past. If all the schools in the country could give to their pupils that thorough training in basic mathematics the Army and the Navy require, several weeks of training now devoted to a review of essential elementary principles could be spent to much better advantage. Also it is true that hundreds of boys of average ability, capable of learning elementary mathematics if given time to absorb the material slowly but unable to keep up with the faster pace required by the emergency of war-time needs, would be able to qualify for those places where they would be of greatest service to the country.

And what of the years after the present war is won? Will mathematics become of less importance as the country gradually returns to peace-time production and peace-time pursuits? I am of the opinion that the need for mathematics will increase in the years after the war rather than decrease. In fact the subject of mathematics should enjoy its "Golden Age."

When one considers the tremendous back-log of buying power in this country now dormant because of cessation of peace-time production; when one bears in mind all of the technical and scientific advances made since the beginning of the war, advances waiting to be applied to peace-time needs; when one realizes the vast reconstruction projects to be undertaken all over the world, projects in which this country must take a leading part; when one considers the millions of people throughout the world to be fed, clothed, and supplied with the necessities of life—one realizes the tremendous scale upon which production in this country must function.

And the same mathematics that is needed to build tanks, fighter planes, and guns, will be needed to build automobiles, transport planes and electric refrigerators. New scientific developments will increase the demand for men and women scientifically trained. And the tax-payers themselves, who in the decade of the twenties could see little practical value in mathematics, have now become educated by "Refresher Books," magazine articles, radio skits and movie shorts to the point where they will demand that their sons and daughters receive substantial courses in mathematics in elementary and secondary schools.

All of these things presage an important role for mathematics in the education of the future. Furthermore all of the essential needs for mathematics at the elementary and secondary school levels during the present emergency and in later years, can, I believe, be met within the framework of the general principles previously stated. We, as teachers of mathematics can make this subject, along with English, a stabilizing force around which the curriculum of the first twelve years is built. The mathematics of these years is now and will be basic in character. Specialization comes in later years. We can built in these years before college a solid foundation provided we stop playing at teaching and really teach, and provided we stop dissipating our energies in the search for trick methods of teaching which demand little real effort on the part of pupils or teachers. It is my firm conviction that no such trick methods exist.

In closing I would like to say a very few words much more general in nature. It is clear to all of us, I am sure, that courses in mathematics and science, no matter how thoroughly and skillfully taught, will not in themselves be adequate preparation for meeting the problems of the post-war world.

These problems will be so varied and complex that the utmost of patience, good judgment, tolerance, and vision will be required in their solution. To deal intelligently with these problems the young men and women of the present generation must have the broadest kind of an education. Through the study of the history of our own country and of its literature, they must acquire a real understanding of the ideals for which this country stands. Through the study of the history and literature of other countries these young people of today must become familiar with the aims and aspirations of other peoples. The present generation must have minds stimulated by all those things implied in our conception of a liberal education in its broadest sense.

If the world of tomorrow is to be that "better world" which we hope it will be, the new generation must acquire a breadth of vision wider than that of the present generation; the new generation must have before it vistas more expansive than those enjoyed by past generations; the ideals of the new generation must be more solidly founded in the teachings of Christ than have been those of our generation.

These are things we can not teach. But if we can teach the young people in our classes to think straight, to cherish justice and fair play; if we can start them on the road which will lead ultimately to a better world—we shall have made some small contribution towards the attainment of that new world.

Arithmetic and the Defense Worker

By Mary Ann Woodard
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National interest, because of the war emergency, has been focused upon the problem of finding personnel, for both industry and the armed forces, who are competently trained in mathematics. Many army and naval officers, industrialist and educators have voiced the common complaint that the training given young Americans in mathematics is inadequate. They are unanimous in their opinion that one great and widespread deficiency lies in the almost universal weakness in the fundamental skills of arithmetic.

The paucity of available data supporting these statements stimulated this investigation. Moreover the writer hoped that some information which would be helpful in improving arithmetic instruction would be found.

No data were available from either the army or navy but a large defense company, the Sperry Gyroscope Company, Inc., cooperated in this investigation by placing at the writer's disposal the data used in this study. These data consisted of the age, sex, education, score on an intelligence test and the results on an arithmetic skills test.[1] The arithmetic skills test was given for diagnostic purposes because the company knew that applicants selected for training needed to apply these skills in their work. At the end of a week's training period a comparable form of the arithmetic-skills test was given to determine the gain in arithmetic-skills status. Both the test items missed and the total scores were available.

The Test

The arithmetic-skills test given included the four fundamental operations with integers, fractions and decimals. There were other items on per cent and mensuration which were not given as these items did not apply. There were two parts, *A* and *B*, to the test which contains almost duplicate items. Part *B*, however, might be considered slightly more difficult in places as it had larger place numbers. Applicants were given only part *A* but the training school group was given both parts, *A* and *B*.

Population

The above data were obtained in the fall of 1942 and the writer used as cases all the October and November applicants who were given the arithmetic-skills test for whom there was complete data available. As this gave between 350 and 360 cases of each sex, the writer arbitrarily chose to equalize the group to 360 of each sex. The cases added were obtained by taking the necessary number from the first available records. It is regrettable that there are only 60 women and 31 men in the group studied after the training period. This group is much smaller as these were the only available cases for whom there was complete data.

The training school group which was studied for gains was a more selected group. Obviously the company had selected them from the total applicants on some basis. Applicants making the lowest intelligence test scores were not assigned to machine tool operation training. Other factors such as the applicant's previous employment record and his comprehension of mechanical principles, his arithmetic-skills test score were also given some weight in the final decision as to whether or not the applicant was selected for this particular type of training.

The Training School

At the time these tests were given, machine tool trainees of the Sperry Gyroscope Company, Inc., spent their first week of

[1] Achievement Test Foundation Skills in Mathematics. By H. C. Christofferson, Carmille Holly Rush, and W. S. Guiler.

employment in a course of subjects related to machine tool operation. Of the 48 hours of training, approximately 20 hours were allotted for shop mathematics, with particular emphasis on fractions and decimal fractions which are used in machining measurements. The purpose was to develop a mastery of the necessary skills if possible in the allotted time. If the employee had sufficient arithmetic skill so that he could grasp the concepts, the right angle trigonometric ratios were taught. The tangent ratio is a very essential idea and one the training school wished to teach but the lack of arithmetic skills and the short period of time given for training made it impossible to teach this principle to many trainees. A shop mathematics work book was used.

Dr. John H. Coleman, Assistant to the Training Director, made the following statement:

To facilitate instruction, trainees were grouped according to their mastery of the fundamental arithmetic skills. The training school teachers were competent shop people, who had completed a 28 hour War-Time course in teaching methods. All had taken the Job Instructor Training course offered by Training Within Industry. Each teacher is put in charge of not more than *12* students per week, who were grouped on the basis of their test performance.

Before the trainees were grouped according to their mastery of fundamental arithmetic skills, those trainees better grounded in mathematics lost interest in the course because of the slow pace. Hence, they tended to be critical of the entire training program.

It proved to be practicable to group the trainees quickly and efficiently. The tests were given immediately following the fifteen minute introductory period. Each instructor scored his own tests. The instructors met promptly and reassigned class members, taking into account for each individual the number of problems missed because of errors in calculations as contrasted with lack of understanding of the processes involved. Somewhat unexpectedly, the instructors were as eager to teach one section as another. They found the ability-sectioning markedly helpful in meeting their instruction problems. It should be added that these instructors were vitally interested in their new positions, eager to learn, and initiated many teaching improvements.

The following table gives a summary of the arithmetic-skills test results of the 720 adult applicants for defense work whose records were studied. The writer believes that an examination of these results should be convincing evidence that the arithmetic-skills ability of the applicants for defense work is far below a reasonable standard of efficiency. While it is not the writer's purpose to establish what adult performance should be on this test it is obvious that workers who found the operations taken into consideration necessary in their daily work would not be capable of calculating them efficiently.

PER CENT OF TEST ITEMS NOT PASSED BY 720 ADULTS ON INITIAL TEST OF 15 ARITHMETIC-SKILLS

PART A

Test Item	Men $N=360$	Women $N=360$	Total $N=720$
Integers			
1. Addition	20.3	26.4	23.3
2. Subtraction	12.2	14.2	13.2
3. Multiplication	35.6	42.8	39.2
4. Division (1 place divisor)	33.1	32.5	32.8
5. Division (2 place divisor)	66.7	66.7	66.7
Fractions			
6. Addition	56.1	54.7	55.4
7. Subtraction	72.5	82.8	77.6
8. Multiplication	73.6	80.0	76.8
9. Division	82.8	82.8	82.8
10. Changing a mixed number to an improper fraction	65.3	61.1	63.2
Decimals			
11. Addition	34.2	25.8	30.0
12. Subtraction	35.3	37.2	36.3
13. Multiplication	21.4	15.8	18.6
14. Division	63.6	65.6	64.6
15. Changing a common fraction to a decimal	84.4	89.2	86.8

These results may be particularly significant in view of Admiral Nimitz's statement that the highest attainments in arithmetic were from recruiting stations in New York State, one of which was Brooklyn. Since these adults come from one of those areas having the highest arithmetic competence, it may not be unreasonable to assume that these results are much above the arithmetic-skills achievement of the country as a whole.

The results were studied for sex differences and though the women as a group might be considered superior to the men

since they were younger and had a higher mean intelligence test score, the statistical analysis gave no reason to reject the hypotheses that they could be considered one homogeneous population or that errors are independent of sex.

This investigation also attempted to determine the relation of age, intelligence and years of school training to arithmetic-skills status.

The results seem to indicate that the only relation of importance is that between arithmetic-skills status and intelligence, which yielded a correlation of .382. While there was some correlation between years of school training and arithmetic-skills status, $r \equiv .296$, this was appreciably lowered when intelligence was held constant. Furthermore the correlation of years of school training with intelligence for this group was .456 which indicates that the more intelligent tended to remain in school longer.

If one makes the assumption that age measures the distance from initial learning and the results are interpreted with this assumption in mind the older adults might be expected to make lower scores. This would be the case if we accept the "disuse" theory as the reason for low arithmetic-skills scores. On the other hand the older ones could have had more experience applying these skills in real situations where they are necessary so might be more proficient in arithmetic-skills. An inspection of the results shows no support for either interpretation. If these influences exist they apparently counteract each other.

Many people particularly older adults insist that they were given more rigorous arithmetical training in "their" day and that school training in the three Rs has deteriorated, and is now sugar-coated or soft. If this were a true statement of actual fact, the older adults should make relatively better arithmetic-skills scores than the younger adults. This study found no evidence that this criticism of the schools is justified.

The most important finding of this study is the gains made in arithmetic-skills status by the training school group after only 20 hours study of shop mathematics. The mean gain of the group on Part A (15 items) was 3.25 and on parts A and B, (30 items) was 6.74. The mean score of the group after training was 27.14 on the 30 items.

As the original test showed that some of the skills were better mastered than others the test results were analyzed to find the per cent of gain on each test item. For this analysis 215 cases were used as 124 previous trainee results were added to the 91 cases. The following table shows the gain made on each item. (Failed on first test, passed on second test.)

1. 18.6%	9. 49.3%
2. 12.1%	10. 29.3%
3. 25.6%	11. 15.8%
4. 25.6%	12. 14.9%
5. 34.4%	13. 9.8%
6. 29.8%	14. 44.2%
7. 55.3%	15. 59.5%
8. 46.5%	

The correlations of gains with age, intelligence and years of school training while small are interesting. They are:

Intercorrelations of Gains in Arithmetic-Skills Scores of The Training School Group

		Intelligence Test Score	Years of School Training	Age
Gain in Arithmetic Skills Score	15 Items	−.012	.004	.062
	30 Items	.029	−.058	.210

These results would lead to the conclusion that given average intelligence a group can be quickly trained to the necessary competence in arithmetic-skills. Moreover they seem to give undeniable proof that these skills have been learned and merely forgotten by a large number and only a refresher course is needed to revive them. If this is the correct interpretation of these results we can say that the schools are teaching the arithmetic-skills but should make more provision for

maintaining them. On the other hand the industrial training school provided a perfect learning situation which the schools cannot hope to equal. The trainees because of the immediate need to use the skills had the essential drive to learn them. The teaching was excellent, the material was presented in relation to its use, and the trainees were mature enough to grasp its significance.

The writer believes that industry's training schools may contribute much information, which will aid in revising our school curricular and instruction.

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Student Attitude As a Factor in the Mastery of Commercial Arithmetic

By Albert L. Billig
Allentown, Pa.

In as much as teaching implies a favorable learning situation it is self-evident that when the content to be mastered exceeds the potential ability of the pupil the learning situation is definitely unfavorable. In the main, the great majority of students have this potential ability but for some reason or other do not avail themselves to the extent which would seem warranted. In an empirical manner, it was observed that the effort which the pupils were willing to exert in order to learn specified content material varied greatly. It was further noted that the effort varied, to a significant degree, according to the expressed attitude of the pupil in question. Accordingly the question posited was: Is student attitude a factor in the mastery of commercial arithmetic?

Newcombe (2) summarizes attitude as "primarily a way of being 'set' toward or against certain things. Both the response and the situations are in most cases of verbal nature—almost exclusively so in so far as attitudes lend themselves to measurement." According to Stagner (3)

> The attitude is characterized always by (1) an object, (2) a direction and (3) intensity. The object may be considered the intellectual or cognitive aspect of the experience; the direction is given by the predominantly pleasant or unpleasant feeling-tone bound up with this intellectual understanding. Intensity may be thought of as related to excitement, or degree of activity which will be released by situations involving the attitude.

During the school year 1937–38 an investigation was conducted involving tenth grade girls at the Hunsicker Annex of the Allentown High School. The attitudes were obtained from essays submitted during a course in commercial arithmetic. Each pupil submitted essays on the following topics: What I think of commercial arithmetic; and, How I feel about arithmetic. An interval of six weeks was allowed to elapse between these essays. They were written as a project, ostensibly to incorporate feasible suggestions to be utilized with later classes. The pupils were encouraged to record just how they felt. They were given assurance that the essays would not be read during the current school year. The original 136 cases were reduced to 108 cases. The reason for this was that twenty students dropped out of school during the year to enter employment. Eight additional cases were dropped, largely, because their intelligence quotient was less than 92 which was assumed to be the minimum needed to pass the course in question, at that time.

As the next school year progressed the usable data was culled from the essays which had been collected. Those statements which indicated the pupils' attitudes were typed on individual cards, with the name of the author on the reverse side. A few representative statements, from the essays, follow:

Some type of problems I enjoy doing, others I just do because I have to.

To my mind mathematics is the most vital, important, and interesting subject.

It is utterly impossible to master this subject, of all the subjects there are.

I think it is a subject worth knowing about. It is something everyone should be interested in.

Many pupils do not like mathematics, and usually when you do not care for a subject it becomes difficult.

Many students think arithmetic is a waste of time, these students are the ones who do not get along in this particular subject.

I believe arithmetic would be more interesting if it were an elective, especially in high school.

I know for a fact that I am not getting more interested in mathematics because I never did like school and was never interested in arithmetic.

The "term average" earned by the pupil, in commercial arithmetic tests, for the school year was recorded on the appropriate card. The cards were then sorted according to term average so that the students earning the lowest "average marks" were coded "one." Those in the middle group were coded "two," and in like manner, pupils falling in the highest group were coded "three." The resulting distribution, for the "term average," divided the cards so that thirty-eight were in the central group, and thirty-five each in the highest and lowest group. The reason for the thirty-eight cases in the central group was duplication of "term average."

TABLE I
ARITHMETIC "TERM AVERAGES" AND STUDENTS' ATTITUDES

Attitude	Highest Group	Middle Group	Lowest Group	Total
Favorable	30	25	2	57
Uncertain	5	11	6	22
Antagonistic	0	2	27	29
Total	35	38	35	108

Now the cards were sorted into piles according to the attitude indicated: favorable, uncertain, and antagonistic. This sorting was done by various individuals: high school students, parents, and teachers. In the final distribution the fifty-seven statements judged favorable were coded "three," the twenty-two uncertain were coded "two," and the twenty-nine antagonistic were coded "one." The resulting distribution will be found in Table I. From these data, a 3×3 fold distribution, the contingency coefficient was found. In as much as the theoretical limit is .816 and the calculated coefficient was found to be .64 we can correct our contingency coefficient and get a C of .78. *Chi* squared equals 74.3, and n equals 4. This indicates an association which could not conceivably have arisen by chance.

From these 108 experimental cards, containing statements indicating pupils' attitudes, a scale was developed containing five intervals. The first interval contained those having the most antagonistic expression of attitude, and the fifth, those expressing the most favorable attitudes. The criteria to determine in which a particular statement was to fall was that of discernible difference, as sensed by the judge ranking the statements. In order to offset personal prejudice, of any particular judge, these cards were ranked by fifteen individuals. A tabulation was kept to note the various ranks a particular statement held during the various sortings. In case a statement received three or more different ranks, it was dropped from further consideration. In this manner the final items were those which tended to consistently hold a particular rank. The final scale consisted of sixteen statements. Three of these statements held rank number five, these being the most favorable. Two held rank number four, or favorable. In the median rank there were three statements, or the so-called neutral individual. Four statements held rank number two, these representing those pupils who dislike the subject. In the lowest rank, number one, there were three statements. These are indicative of the antagonstic student. Due to the manner in which ranking was made, it was possible to assign scale values directly, as they were similar. Rank and assigned scale values were assumed to be equivalent, after Linkert (1). "His research had clearly shown that degree-of-agreement responses (e.g., strongly agree, agree, uncertain, disagree, and strongly disagree) to attitude statements could be scored by simply assigning values 1 to 5, . . . " (2). The scale follows: (Copyrighted in 1942)

Directions: Check the statements which describe how you feel about arithmetic. Mark at least two statements.

1. I like it the best of all the subjects. (5)
2. I feel that arithmetic is good for those who like it. (3)
3. It should be an elective. (3)
4. It is the most impossible subject there is. (1)
5. Arithmetic is a good study and can be used by most pupils. (4)
6. I dislike arithmetic; I always did and always will. (1)
7. It is necessary in our everyday living. (4)
8. I do arithmetic only because I must. (2)
9. In my opinion, arithmetic is the most important subject taught. (5)
10. I am not interested in arithmetic. (2)
11. I feel a definite satisfaction in being able to use it. (5)
12. It only gives me a headache. (1)
13. I will only do it if I cannot get out of it. (1)
14. We should be able to take it if we want to, but not be compelled to. (3)
15. The difficult arithmetic is good for those who wish to go to college, but it is of no use to the rest of us. (2)
16. We only use simple arithmetic; the difficult arithmetic we forget shortly after we have had it. (2)

SCORING: Find the mean of the "weights" of the statements marked. Means of five to four, very favorable attitude. 3.99 to 3; favorable attitude. 2.99 to 2; indifferent or somewhat antagonistic attitude. 1.99 to 1; unsatisfactory attitude.

The numbers, in parenthesis, following each statement are their "attitude weight."

During the past school year, 1942–43, the scale was administered at the end of the first rating period, which is essentially six weeks after the opening of school in September. Those pupils securing scale values indicating an indifferent attitude toward the subject were interviewed shortly after the scale was administered. This interview served to ascertain what probable conflicts existed in the mind of the pupil. Some pupils changed to a more favorable attitude as attested by greater effort and higher achievement. It is true, others in this indifferent category persisted at the same level. For pupils with an antagonistic attitude the prognosis is indeed unfavorable, but, just for the record, attempts were made to ameliorate the condition. Pressure was countered by overt resistance, or complete docility. Since no facilities existed to transfer these pupils to another course a down-grading, of the difficulty and the amount, of content material was necessary. This, of course, presaged failure of the pupil in this course. However, a beginning was made to rechannelize his attitude by allowing him to experience success, or at least preventing repeated failure caused by the emotional disruptive approach hitherto held.

In conclusion it can be said that the developed scale proved satisfactory within the limits for which it was designed. It served as an economical method of bringing to the fore those pupils who were most likely to experience failure in commercial arithmetic due to their point of view, namely, an indifferent or an antagonistic attitude toward the material to be mastered. On the positive side it suggested the pupils who would profit most by additional supplementary work. The teaching situation resolves itself, finally, to maintaining the favorable attitude evidenced by the majority of pupils, and the rechannelization of those holding an indifferent point of view. This rechannelization is most likely to be accomplished by tying the present material with their intended life work, the work at which they intend to earn their livelihood, thereby giving it economic potency. The antagonstic pupil has to have his work down-graded. The immediate utility of this down-graded work, in everyday living, occasionally brings a marked change in attitude. If the pupil does not gain insight, failure still occurs. On the whole, the pupil's attitude is a significant factor in determining his probable mastery of commercial arithmetic.

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The Wind Star of Air Navigation

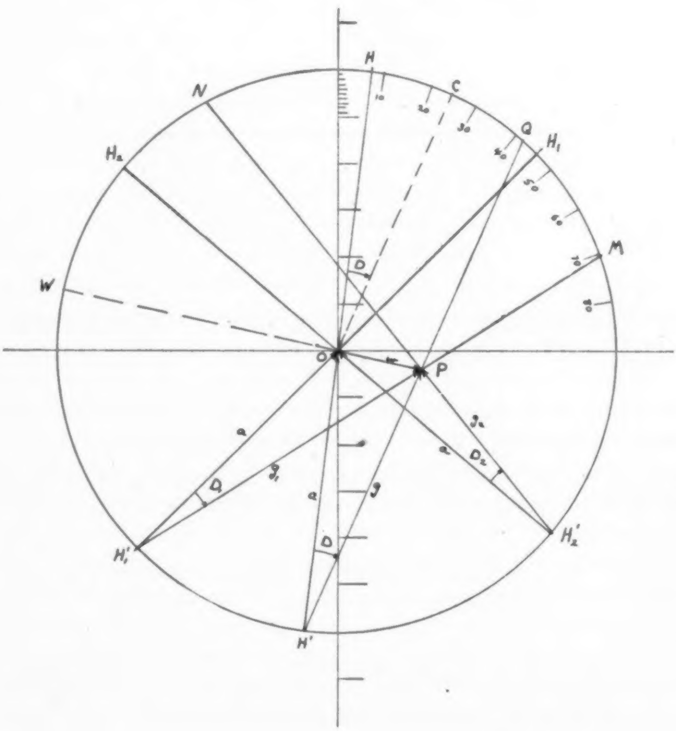
By Ralph Calvert
Utah State Agricultural College, Logan, Utah

The method for the construction of the wind star given herein differs from the usual method in that the radius of the compass rose is taken to represent the airspeed, and the drift angle is constructed by making use of the fact that an angle with its vertex on a circle is measured by one-half the arc intercepted on the circle. None of the books on the mathematics of aircraft navigation that I have had an opportunity to examine has given this construction.

Course plotting paper or polar coordinate paper may be used.

Let H_1 and H_2 be two trial headings with respective drift angles D_1 and D_2. Here it is assumed that both drift angles are to the right. With these four quantities and the airspeed a known, the direction of the wind W and the speed of the wind w may be found as indicated below. If in addition a desired course C is given, the heading H required to make good that course and the drift angle D for this heading may be found. All headings and courses used herein are assumed true.

Through the center O of the compass rose draw the line H_1H_1' where H_1 is the first trial heading and H_1' is the reciprocal heading. To the heading H_1 add twice the drift angle D_1 (if the drift is to the left, subtract) and thus locate the point M. Draw the line $H_1'M$. Draw H_2H_2', add twice D_2 to H_2 and thus locate the point N. Draw $H_2'N$ which intersects the line $H_1'M$ at the point P. Then OP is the wind vector. Extend OP backwards through O to cut the compass rose at W where the direction



of the wind may be read. Since the radius of the compass rose represents the airspeed, the speed of the wind may be found by measuring OP in the units indicated along the vertical axis and solving the proportion $a/60 = w/OP$ for w.

If it is desired to make good a course C, draw PQ parallel to OC and extend to cut the compass rose at H'; draw $H'O$ and extend to cut the compass rose at H where the required heading may be read. The arc HC measures the drift angle D. As a check arcs HC and CQ are equal. The ground speed g may be found by measuring the distance $H'P$ in the units indicated on the vertical axis and solving the proportion $a/60 = g/H'P$ for g.

If a third trial heading and corresponding drift angle are plotted, the ground speed vectors will form a small triangle at the point P. The center of this triangle is then taken as the head of the wind vector.

This method assumes that the compass rose is large enough so that its radius may be used to represent the airspeed and give a distance scale large enough for required accuracy.

In the drawing above the complete degree scale has been omitted for the sake of simplicity.

NEW VISUAL TEACHING AIDS SUPPLEMENT MATHEMATICS, AIR AGE PHYSICS (MECHANICS) AND DRAFTSMANSHIP INSTRUCTION

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◆ THE ART OF TEACHING ◆

Errors in Ninth Grade Algebra*

By Everett Snider
Pikeville, Ky.

The purpose of this study was to attempt to determine (1) the classes of errors most likely to occur in ninth grade algebra and (2) whether or not it is better to follow the rules as presented in most texts for algebraic addition and subtraction or to omit the rules entirely.

Forty-eight ninth grade students in the Taylorsville High School, Taylorsville, Kentucky, were divided into two groups of twenty-four students each. The two groups were equated as to their IQ, past grades in arithmetic, teacher, physical conditions, and text. The two groups were taught by a combination of the individual instruction method, the group instruction method, and daily recitations and assignments. Both groups received as nearly as possible the same type of instruction with the exception of the period spent in mastering the unit on directed numbers. Group I used the rules while learning to add and subtract and at the same time they were taught the meaning of the operations as was the second group. Group II was taught addition and subtraction by the use of the number scale, the thermometer, directions, profit and loss, weights, and by counting; but the rules were never used in class. Review of the two operations was carried out in each group in the same manner as the group had first studied the unit.

For the purpose of obtaining results for the first phase of the study, tests that had been prepared by the teacher were given at the completion of each unit to both groups at approximately the same time. A standardized test was given at the end of each semester. The errors made were classified into sixteen classes. To obtain results for the second phase of the study, a mastery test of addition and subtraction problems was given each group. Each day for the next ten days, while studying multiplication and division of directed numbers, addition and subtraction was reviewed from ten to fifteen minutes; the same test was then given each group again. After three months, without further special remedial instruction, the same test was again given.

There was a total of 1343 errors on the unit tests and on the semester tests. The five largest groups of errors and the corresponding percentages were as follows: Forming equations from verbal problems, 17 per cent; addition of directed numbers, 13 per cent; subtraction of directed numbers, 11 per cent; multiplication of directed numbers, 9 per cent; and substitution, 9 per cent. The remaining 41 per cent was divided into eleven classes.

The total errors in addition for Group I on the test, the two retests, and the errors made on the unit tests and semester test which followed, was 50 − per cent of the total errors. The total errors for Group I in subtraction on the same tests was 59 + per cent.

The errors for this study were finally taken from the ten students of one group that most nearly paired with ten of the other group. Because of the small number of cases, the study definitely shows nothing; but there were some tendencies that were indicated. (1) The most frequent

* Based on an unpublished M. A. thesis done at the Colorado State College of Education, Greeley, 1938.

difficulties were encountered in this order: Formation of equations, addition, subtraction, multiplication, and substitution; (2) errors in multiplication and division of directed numbers increased relatively throughout the year; (3) the number of errors varied directly with intelligence; (4) in addition and subtraction of directed numbers, students tend to have less difficulty if the unit is studied without the use of rules, especially is this true of subtraction; and (5) weaknesses in the four fundamental operations of algebra and in arithmetic tend to be important causes of difficulty with algebra.

It is recommended that teachers lay a solid foundation in the four fundamental operations of directed numbers, that teachers check students closely on arithmetical errors, that meaning and understanding be given to verbal problems, and that addition and subtraction of directed numbers be taught and studied without the use of rules.

WASHINGTON, D. C., February 28, 1944. The American Council on Education has just announced the appointment of a Commission on Motion Pictures in Education. The present members are: Mark A. May, chairman; George S. Counts; Edmund E. Day; Willard E. Givens; George Johnson; George F. Zook, ex officio. The work of the Commission is supported by a grant from eight motion picture production companies made through the Motion Picture Producers and Distributors of America, Incorporated. The eight contributing companies are: Columbia, Loew's Incorporated (M.G.M.), Paramount, R. K. O., Twentieth-Century Fox, United Artists, Universal, and Warner Brothers. The grant covers a five year period.

The Commission will study the needs of schools and colleges for motion picture material and will plan for the production of new films for courses of study in which new pictures are needed. Special attention will be given to the planning of series of films for educational activities connected with postwar reconstruction. The Commission invites the cooperation of all interested educators and educational groups. Suggestions concerning needed productions for educational purposes will be welcomed. The Commission is particularly interested in receiving curriculum materials that can be used as the basis for films. As fast as these materials can be put into shape for filming and approved by competent educational consultants, they will be distributed to all interested producers. For the time being, all inquiries should be addressed to the Chairman, Mark A. May, 28 Hillhouse Avenue, New Haven, Connecticut.

EDITORIALS

The New President of the National Council of Teachers of Mathematics

Dr. F. Lynwood Wren, Professor of Mathematics at George Peabody College for Teachers at Nashville, Tennessee, is the newly elected President of the National Council of Teachers of Mathematics.

Dr. Wren, a native Tennessean, received the A.B. degree, *optime merens*, in 1915 from the University of the South. In 1925 he received the M.A. degree in Educational Psychology from George Peabody College for Teachers. In 1930 he received the Ph.D. degree in Mathematics from the University of Chicago. He also did graduate work in Mathematics at Johns Hopkins University in 1917 and again in 1921.

He taught in City High School, Martin, Tennessee; in the McFerrin School and in The McCallie School for boys at Chattanooga, Tennessee. He served as private and then sergeant in the Quartermaster Corps of the U. S. Army between December, 1917 and June, 1919. He saw foreign service in France. He was appointed Julia A. Sears Associate Professor of the Teaching of Mathematics at Peabody in 1927 and became Julia A. Sears Professor of Mathematics in 1930. He has served as Vice-President of the Tennessee Academy of Science in 1939 and President in 1940; Vice-President of the National Council of Teachers of Mathematics, 1939–1941; member of the Executive Committee of the Southeastern Section of the Mathematical Association of America. He is co-author of the *Number Readiness Series of Arithmetics* and *The Teaching of Secondary Mathematics*. He was a contributor to the 16th and 17th Yearbooks of the National Council of Teachefs of Mathematics, the *Encyclopedia of Educational Research*, and numerous magazines.

The Mathematics Teacher wishes to congratulate both Professor Wren and the Council on his election to the presidency and to wish Dr. Wren success in his new work. The task of guiding the selection of content material and the training of teachers for the schools in the post-war world is not an easy one, but Dr. Wren is well qualified to lead the way. W. D. R.

A Message from the New President

To be elected President of the National Council of Teachers of Mathematics is indeed an honor of the greatest magnitude which I fully appreciate. I shall make every effort to meet the responsibilities of the office to the very best of my ability.

Today education is confronted with many very serious problems. Many of us are now busily engaged in varied types of war activities which occupy most of our time and effort. Amidst all of our concern for the present, let us not forget that we should be interested in the problem of what the plan for post-war education is to be. The entire educational program of our schools has been, and still is being, subjected to rather severe shock. The speed-up in teaching and the streamlining of curriculum content which have been recognized as necessities because of the war emergency are going to leave their marks on teaching procedures and content selection and organization. Let us, as teachers of mathematics, not be caught asleep at the switch. There is not one of us who is free from the responsibility of earnest thinking on the problem of the postwar program of mathematics in our schools.

We must also keep uppermost in our minds the current problems which are

ours. It is urgent that we keep our standards high; inexperienced teachers need our help; we must not sacrifice all of the cultural value of our subject for the more practical; we must be careful in making deductions concerning curriculum content and teaching procedures; those practices and that material which are very effective for the more mature mind and in highly motivated situations will not necessarily be good under more normal classroom conditions; the work of the Council should be so organized that it will be carried to teachers of mathematics everywhere in the country. Your Board of Directors has already been giving thought to these problems. They are your problems as well as theirs. If you have ideas which you think can contribute to the welfare of the Council and to the improvement of instruction in mathematics, let us hear from you.

I earnestly seek the cooperation of every member of the Council that we may all direct our best efforts toward making the teaching of mathematics more meaningful, the content of mathematics more significant, and the work of the National Council more effective. F. Lynwood Wren

In Appreciation of President Smith

There is no doubt that former president Dr. Rolland R. Smith has increased the prestige of the National Council of Teachers of Mathematics by his unstinting efforts to improve mathematical education in elementary and secondary schools. While the war made it impossible for the Council to hold its regular annual meetings during the past two years, nevertheless Dr. Smith did not let this fact keep the Council in the background. Through regional meetings the spirit of the organization was kept alive and a going concern and the regional meeting in New York on February 26th is a good example of the kind of spirit and enthusiasm which has been fostered. Largely through the work of Dr. Smith the Council has worked in close touch for the first time in its history with the U. S. Department of Education in Washington through its leaders, Dr. John Studebaker, Giles M. Ruch and others. It is for these reasons that we can say that President Smith has left the Council in a strong position from where it can make still stronger efforts to improve mathematical education. W. D. R.

Concerning Subscriptions that Expire in May

Any one who began his subscription to The Mathematics Teacher with the October issue in 1943 is automatically a member of the National Council of Teachers of Mathematics until October, 1944. However, since no issues of the magazine are published in June, July, August and September of 1944, those who paid the membership fee of $2.00 in October of last year should send in their renewals before October, 1944 in order to save the Council inconvenience and loss of money. Costs of publication are rising and in order not to have to raise the price of the journal during the emergency we bespeak the cooperation of our members in being prompt in making renewals. In order to make sure that this matter is not overlooked, The Mathematics Teacher will send out early in April, if not before, cards to all members whose subscriptions expire in May (even though their membership runs to October) and it is hoped that members will be prompt in renewing membership so as to facilitate matters in the office. It has been almost impossible to plan for the October issue each year because members are so careless about renewing in time. Moreover, entirely too many members fail to renew at all even though two or three notices and a personal appeal from the Editor have been sent out. It is more important now than ever before "stick by the ship" if we are to weather the storms ahead and if the Council is to continue to do its work effectively. W. D. R.

IN OTHER PERIODICALS

By NATHAN LAZAR
Midwood High School, Brooklyn, New York

The American Mathematical Monthly

December, 1943, vol. 50. no. 10.

1. Ulom, S. M., "What Is Measure?" pp. 597–602.
2. Conkwright, N. B., "An Elementary Proof of The Budan-Fourier Theorem," pp. 603–605.
3. Beer, F. P., "A Plane Representation of Vectors and Tensors," pp. 605–610.
4. Fischer, I. C., "A Projective Construction for Plane Nodal Cubics," pp. 611–617.
5. Robbins, H. E., "A Note on the Riemann Integral," pp. 617–618.
6. Brock, J. E., "An Example of Double Series," p. 619.
7. Reiner, Irving, "Functions, not Formulas, for Primes," pp. 619–621.
8. Schwerdtfeger, H., "On Pr-Matrices," p. 621.
9. Frame, J. S., "Solving a Right Triangle without Tables," pp. 622–623.
10. Newson, C. V., "The Navy College Training Program," pp. 645–650.

National Mathematics Magazine

January, 1944, vol. 18, no. 4.

1. Merrill, Lynn L., "Professor James McGiffert," pp. 142–144.
2. Sleight, E. R., "John Napier and His Logarithms," pp. 145–152.
3. Betz, William, "Next Steps in Education and in the Teaching of Mathematics," pp. 153–176.

School Science and Mathematics

February 1944, vol. 44, no. 2.

1. Gingery, Walter G., "Map Projections for an Air Age," pp. 101–111.
2. Davis, H. T., "Archimedes and Mathematics," pp. 136–145.
3. Johnson, John T., "Can Concepts in Elementary Mathematics Be Developed?" pp. 146–154.
4. Nyberg, Joseph A., "Notes from a Mathematics Classroom," (continued) pp. 155–158.

Miscellaneous

1. Asbury, F. C., "Points and Squares," *School* (Secondary Edition), 32: 231–236, November, 1943.
2. Brueckner, L. J., "Improving Mathematical Abilities of Pre-Induction Groups," *Bulletin of the National Association of Secondary-School Principles*, 27: 33–48, December, 1943.
3. Fehr, H. F., "Mathematics in Pre-Induction Training," *Teachers College Record*, 45: 161–168, December, 1943.
4. Greenfield, S. C., "Saving Repeaters in Geometry," *High Points*, 25: 51–53, December, 1943.
5. Hartung, M. L., "Significant Analysis of Mathematical Needs," *School Review*, 51: 516–518, November, 1943.
6. Hawkins, R. H., "Developing the Quadratic Equation," *School* (Secondary Edition), 32: 428–429, January, 1944.
7. Hone, P. W., "Instructions in How to Study Geometry," *School* (Secondary Edition), 32: 329–331, December, 1943.
8. Hunter, M. W., "Arithmetic for Beginners," *The Instructor*, 53: 14–15, January, 1944.
9. Kidd, H. C., "Overcoming Difficulties in Mathematics," *School* (Elementary Edition), 32: 427–429, January, 1944.
10. Mannello, Jr. G., "Do You Know Your Fractions," *The Grade Teacher*, 61: 46+, January, 1944.
11. Mast, G. N., "Reading Nomographs or Alignment Charts," *Industrial Arts and Vocational Education*, 32: 421–422, December, 1943.
12. McDaniel, L., "Mathematics in a Secondary School for Girls," *Southern Association Quarterly*, 7: 460–467, November, 1943.
13. Metcalfe, L. S., "New Visual Aids for Mathematics," *Secondary Education*, 11: 11–13, November, 1943.
14. Richardson, R. G. D., "Mathematics and the War," *American Scholar*, 12: 503–505, October, 1943.
15. Salkind, Charles, "Placing the Decimal Point," *High Points*, 25: 66–67, December, 1943.
16. Schlotzhauer, F. H., "Drill Devices in Arithmetic," *School* (Elementary Edition), 32: 327–329, December, 1943.
17. Schwamm, P., "Mathematics and the Muses," *Journal of Education*, 126: 293, December, 1943.
18. Scott, A. R., "Device Used in Teaching Trigonometry," *School* (Secondary Edition), 32: 336–337, December, 1943.
19. Shaw, R. B., "Experiment in the Use of Goal Sheets in Ninth Grade Mathematics," *Journal of Educational Research*, 37: 209–211, November, 1943.
20. Todd, J. M., "Making a Mathemztical Mural," *Wisconsin J ournal of Education* 76: 153–154, November, 1943.
21. West, P. V., "How to Make Graphs," *Nations' Schools*, 32: 56+, December, 1943.

NEW BOOKS RECEIVED

Almstead, Francis E. and Tuthill, F. R. I., RADIO MATÉRIEL GUIDE. McGraw-Hill Book Co., 1943. 242 pp. Price, $2.00.

Ballou, Donald H. and Steen, Frederick H., PLANE AND SPHERICAL TRIGONOMETRY WITH TABLES. Ginn and Co., 1943. 179 pp., 84 tables. Price, $2.40.

Bradley, A. D. and Upton, Clifford B., AIR NAVIGATION WORKBOOK. American Book Co., 1943. 112 pp. Price, $0.88.

Cooke, Nelson M. and Orleans, Joseph B., MATHEMATICS ESSENTIAL TO ELECTRICITY AND RADIO. McGraw-Hill Book Co., 1943. 418 pp. Price, $2.40.

Freilich, Aaron, Shanholt, Henry H. and Seidlin, Joseph, SPHERICAL TRIGONOMETRY. Silver Burdett Company, 1943. 140 pp. Price, $1.28.

Harding, Lowry W., ARITHMETIC IN ACTION. Ohio State University Press, 1943. Mimeographed. 109 pp. Price, $1.00.

Harding, Lowry W., ARITHMETIC THROUGH EXPERIENCE. Ohio State University Press, 1943. Mimeographed. 103 pp. Price, $1.00.

Hartley, Miles C., TRIGONOMETRY. Plane and Spherical, Enlarged Edition. The Odyssey Press, 1943. 328 pp. Price, $1.60.

Kells, Lyman M., CALCULUS. Prentice-Hall, Inc., 1943. 508 pp. Price, $3.75.

Kells, Lyman M., Kern, Willis F., and Bland, James R., NAVIGATION. McGraw-Hill Book Co., 1943. 479 pp. Price, $3.75.

Kells, Lyman M., Kern, Willis F., and Bland, James R., PLANE AND SPHERICAL TRIGONOMETRY. McGraw-Hill Cook Co., 1943. 400 pp. Price, $2.00.

Kern, Willis F. and Bland, James R., GEOMETRY WITH MILITARY AND NAVAL APPLICATIONS. John Wiley and Sons, Inc., 1943. 152 pp. Price, $1.75.

Lebowitz, Samuel H., PRE-SERVICE COURSE IN MACHINE SCIENCE. John Wiley & Sons, Inc., 1943. 440 pp. Price, $2.50.

Leighton, Henry L. C., SOLID GEOMETRY AND SPHERICAL TRIGONOMETRY. D. Van Nostrand Company, 1943. 210 pp. Price, $2.20.

Mallory, Virgil S., MATHEMATICS FOR VICTORY. Benj. H. Sanborn & Co. 430 pp. Price, $1.68.

Naidich, James, MATHEMATICS OF FLIGHT, McGraw-Hill Book Co., 1943. 409 pp. Price, $2.20.

Nelson, Alfred L. and Folley Karl W., PLANE AND SPHERICAL TRIGONOMETRY, Revised Edition, Harper and Bros., 1943. 247 pp., 134 pp. of tables. Price, $2.40 with tables, $1.80 without tables.

Northrop, Eugene P. RIDDLES IN MATHEMATICS. D. Van Nostrand Co., 1944. 262 pp. Price, $3.

Reeve, W. D., ESSENTIAL MATHEMATICS. The Odyssey Press, 1943. 284 pp. Price, $1.32.

Richtmeyer, Cleon C. and Foust, Judson W., BUSINESS MATHEMATICS. McGraw-Hill Book Co. 1943. 401 pp. Price, $2.75.

Rider, Paul R. and Hutchinson, Charles A., NAVIGATIONAL TRIGONOMETRY. The Macmillan Co., 1943. 232 pp. Price, $2.00.

Schorling, R., Clark, J. R. and Lankford, Francis G. Jr., STATISTICS. World Book Co., 1943. 76 pp. Price, $0.44.

Schnell, Leroy H. and Crawford, Mildred, CLEAR THINKING (Rev. Ed.) Harper & Bros., 1943. 358 pp. Price, $1.96.

Steen, Frederick H. and Ballou, Donald H., ANALYTIC GEOMETRY. Ginn and Company, 1943. 206 pp. Price, $2.40.

Stelson, Hugh E. and Rogers, Harold P., COMMERCIAL ALGEBRA. The Macmillan Co., 1943. 283 pp. Price, $2.50.

Theissen, A. D. and McCoy, Louis A., PLANE GEOMETRY, Loyola University, 1943. 344 pp. Price, $1.40.

Waugh, Albert E., ELEMENTS OF STATISTICAL METHOD, McGraw-Hill Book Co., 1943. 532 pp. Price, $4.00.

Weeks, Aithur W. and Funkhouser, H. Gray, PLANE TRIGONOMETRY. D. Van Nostrand Co., 1943. 193 pp. Price, $1.75.

Whitaker, George H., BUSINESS MATHEMATICS. McGraw-Hill Book Co., 1943. 185 pp. Price, $1.50.

State Representatives of the National Council of Teachers of Mathematics*

Alabama: Mrs. J. Eli Allen, Phillips High School, Birmingham.
Arizona: Miss Myra R. Downs, 93 West Culver St., Phoenix.
Arkansas: To be filled.
California: Miss Emma Hesse, University High School, Oakland.
Colorado: Mr. H. W. Charlesworth, East High School, Denver.
Connecticut: Miss Dorothy S. Wheeler, Bulkeley High School, Hartford.
Delaware: Mr. Harry E. Algard, Jr., Tower Hill School, Wilmington.
Dist. of Col.: Mrs. Ethel Harris Grubbs, 751 Fairmont St., Washington, D.C., and Miss Veryl Schult, Wardman Park Hotel, Washintgon, D. C.
Florida: Dr. F. W. Kokomoor, University of Florida, Gainesville.
Georgia: Miss Bess Patton, 1585 N. Decatur Rd., N. E. Atlanta.
Idaho: To be filled.
Illinois: Dr. Miles Hartley, University High School, Urbana.
Indiana: Dr. L. H. Whitcraft, Ball State Teachers College, Muncie.
Iowa: Miss Dora Kearney, Iowa State Teachers College, Cedar Falls.
Kansas: Miss Ina Holroyd, Kansas State College, Manhattan.
Kentucky: Miss Dawn Gilbert, 1331 Clay St., Bowling Green.
Louisiana: Miss Jessie May Hoag, Jennings High School, Box 837, Jennings.
Maine: Miss Pauline Herring, 360 Spring St., Portland.
Maryland: Miss Agnes Herbert, 806 E. North Ave., Baltimore.
Massachusetts: Mr. Harold B. Garland, 129 Houston Ave., Milton.
Michigan: Mr. Duncan A. S. Pirie, 950 Selden Ave., Detroit.
Minnesota: Miss Edith Woolsey, 3024 Aldrich Ave. S., Minneapolis.
Mississippi: Mr. Dewey S. Dearman, State Teachers College, Hattiesburg.
Missouri: Mr. G. H. Jamison, State Teachers College, Kirksville.
Montana: Miss Gertrude Clark, 403 Eddy Ave., Missoula.

* The State Representatives of The National Council whose names appear on this page are doing excellent work for the Council. It is hoped that teachers of mathematics all over the country will cooperate with them in their respective localities in order that our membership may be appreciably increased.—The Editor.

Nebraska: Dr. A. R. Congdon, University of Nebraska, Lincoln.
Nevada: Mr. R. van der Smissen, White Pine County High School, Ely.
New Hampshire: Mr. H. Gray Funkhouser, Phillips Exeter Academy, Exeter.
New Jersey: Miss Mary C. Rogers, 425 Baker Ave., Westfield.
New Mexico: To be filled.
New York: Mr. H. C. Taylor, Benjamin Franklin High School, Rochester.
North Carolina: Professor H. F. Munch, Chapel Hill.
North Dakota: Miss Henrietta L. Brudos, State Teachers College, Valley City.
Ohio: Mrs. Florence Brooks Miller, 3295 Avalon Road, Shaker Heights.
Oklahoma: Miss Eunice Lewis, 2203 East 14th St., Tulsa.
Oregon: Mr. Edgar E. DeCou, University of Oregon, Eugene.
Pennsylvania: Dr. Lee Boyer, State Teachers College, Millersville.
Rhode Island: Mr. M. L. Herman, Moses Brown School, Providence.
South Carolina: Miss Harriet Herbert, University of South Carolina, Columbia.
South Dakota: Miss Josephine Wagner, Sioux Falls Schools, Sioux Falls.
Tennessee: Mr. R. C. Shasteen, Clarksville.
Texas: Miss Elizabeth Dice, North Dallas High School, Dallas.
Utah: Mr. R. B. Thompson, University of Utah, Salt Lake City.
Vermont: Mr. John G. Bowker, Middlebury College, Middlebury.
Virginia: Miss Carrie B. Taliaferro, Box 102, State Teachers College, Farmville.
Washington: Miss Kate Bell, Lewis and Clark High School, Spokane, and Mr. S. E. Boselly, Route 9, Box 564N. Seattle.
West Virginia: Miss May L. Wilt, 107 High St., Morgantown.
Wisconsin: Miss Theda Frances Howe, Riverside High School, Milwaukee.
Wyoming: Dr. O. H. Rechard, University of Wyoming, Laramie.

Canada

Alberta: Professor A. J. Cook, University of Alberta, Edmonton.
Manitoba: Mr. D. McLeod, 716 Ingersoll St., Winnipeg.
Nova Scotia: Mr. Warren G. Roome, Amherst Public Schools, Amherst.
Porto Rico: Miss Conchita R. de Lopez, Universidad de Puerto Ricó. Rio Piedras.

NEWS NOTES

1944 SUMMER SESSION
AT TEACHERS COLLEGE, COLUMBIA UNIVERSITY
IN THE TEACHING OF MATHEMATICS

Teachers College, Columbia University, will offer the following courses in the teaching of mathematics in the summer session of 1944 which begins on July 3 and ends on August 11:

> By Professor John R. Clark: Teaching arithmetic in the elementary school; Modern business arithmetic. By Dr. Nathan Lazar: Teaching geometry in secondary schools. By Mr. G. R. Mirick: Elementary mechanics; Observation and participation in the teaching of mathematics. By Professor W. D. Reeve: Teaching algebra in secondary schools; Teaching and supervision of mathematics: senior high school. By Professor C. N. Shuster: Mathematics applied to elementary engineering; Preflight training in Civil Aeronautics Navigation (July 3 to 21); Navigation (July 24 to August 11).

There will be held during the summer session, on consecutive Thursdays beginning on July 6, five informal conferences in which all of the instructors above and other persons from outside will participate for the purpose of bringing before the students vital questions relating to the reorganization and teaching of mathematics in the post-war world. There will be opportunity for discussion in which all of the students will be invited to take part. These conferences have come to be a common meeting place for all students, instructors, and guests, and thus serve both professional and social ends. Registration for these conferences is not necessary and all those who are interested in the improvement of mathematical education are invited to attend.

Special features of the University of Chicago Workshop this summer will be sections on Inter-American Education and Aviation Education. Participants, in addition to receiving help and counsel from consultants especially selected for their competence in these fields, will hear lectures, see films and have access to much new material pertaining to Latin-America and Aviation.

Ralph W. Tyler, Chairman of the Department of Education, is director of the Workshop, which will include sections on Elementary and Secondary Education, and Human Development. Teachers, administrators and librarians who wish help in solving problems in their own classrooms and in adjusting their schools to war and post-war demands will be particularly interested in the offerings in curriculum, guidance, and evaluation.

A limited number of scholarships paying either full or half tuition are available. Further information may be secured by writing to James B. Enochs, Executive Secretary of the Workshop, University of Chicago, Chicago 37, Illinois.

The forty-first annual meeting of The Association of Teachers of Mathematics in New England was held on Dec. 4, 1943 at Boston University. The program follows:

Morning Session

10:15 Business Meeting.
Election of officers and reports.

10:45–12:15 Talks and discussion.

"Readiness in Mathematics—Its influence on practices and procedures in the Junior High School program." MR. CHARLES O. RICHTER, *Division of Research and Guidance, Public Schools, Newton, Mass.*

"The Old Time Religion in Secondary Mathematics." PROF. EVIN A. NASON, *Phillips Academy, Andover, Mass.*

Afternoon Session

2:00 TOPIC: Experiences with Wartime Mathematics.

For the Army: PROF. RALPH BEATLEY, *Harvard University*

For the Navy: PROF. TITUS E. MERGENDAHL, *Tufts College*

For the Civilian: PROF. JOSEPH SPEAR, *Northeastern University*

Dates for future Meetings of the A. T. M. in N.E.

April, 8, 1944—Dinner—Evening
May 6, 1944—All day—Boston

The Association of Teachers of Mathematics in New England and The Connecticut Valley Section of the Association held a joint meeting at Springfield, Mass., on March 4, 1944. The following program was given:

Morning Session

11:00 "The Eighth Grade and the Development of Mathematical Reasoning." MISS ELIZABETH M. MEYER, *John W. Weeks Junior High School, Newton, Mass.*

"Methods of Approximating Square Root." PROF. WILLIAM FITCH CHENEY, JR., *University of Connecticut, Storrs, Conn.*

Afternoon Session

2:00 Annual business meeting of the Connecticut Valley Section.

"The World Calendar." MR. EMERSON BREWER, *Director of the World Calendar Assn., 630 Fifth Ave., New York City*

"Our Responsibility in Teaching Mathematics." PROF. JOSHUA I. TRACEY, *Yale University, New Haven, Conn.*

A joint meeting of The Cleveland Mathematics Club and The National Council of Teachers of Mathematics was held at Hotel Statler, Cleveland, Ohio on Saturday, March 25, 1944.

Program

Morning Session—10:00 A.M.

Panel Discussion: Ninth Year Mathematics.

Address: The Impact of the War on Junior and Senior High School Mathematics. DR. ROLLAND R. SMITH, Specialist in Mathematics, Public Schools, Springfield, Mass.

Discussion Period.

Luncheon—12:00 M.—Hotel Statler

Afternoon Session—1:30 P.M.

Address: The Place of Arithmetic in the Curriculum of the Secondary School. Dr. C. L. Thiele, Director, Exact Sciences, Detroit Public Schools.

Address: Recent Changes in the Tenth Year Course in Geometry. Dr. Rolland R. Smith.

Question Period.

Textbook materials for use in training America's youth for living in "the air age" are being prepared by a research staff of the Stanford University School of Education for the Civil Aeronautics Administration, Washington, D. C.

Under the direction of Paul R. Hanna, professor of education, a staff of 25 persons has started a six-month project which involves gathering material on aviation suitable for inclusion in textbooks and courses-of-study at elementary and junior high school levels.

Stanford was assigned the project after the Civil Aeronautics Administration had been deluged with requests from textbook writers for aviation materials. The volume which will result from the present investigation will be designed to serve the needs of these writers.

Based on the necessity of developing an air-minded generation able to cope with the new problems associated with aircraft, the projected source volume will include aviation material in the arts, arithmetic, guidance and mental health, language arts, science, and social studies.

Suitable curriculum content will be developed for the levels from the first grade through the ninth.

Roscoe B. Bancroft of the C.A.A. staff in Washington has been loaned to Stanford to serve as chief aviation consultant. Mrs. Lorraine Sherer, formerly director of curriculum of the Los Angeles County public schools, is coordinator of research.

Members of the Stanford faculty serving as chief consultants in the various fields include Daniel M. Mendelowitz, arts; Norman Fenton, guidance and health; L. B. Kinney, arithmetic; R. Will Burnett, science; Holland D. Roberts, language arts; and I. James Quillen, social studies.

Dr. Giles M. Ruch, Chief of the Division of Research and Statistical Service, Vocational Division, of the U. S. Office of Education in Washington, D. C., passed away on Monday, November 15, 1943. Dr. Ruch was a member of the recent committee on "Pre-Induction Training in Mathematics," where he made an important contribution. Dr. Ruch was one of the co-authors of a series of well known textbooks published by Scott, Foresman and Company. He was a member of the National Council of Teachers of Mathematics.

Leslie Leland Locke, Professor of Mathematics for twenty-two years at Brooklyn College evening session, died on August 29 of last year. He was a member of the National Council of Teachers of Mathematics. Professor Locke was the author of many articles on mathematics and collected more than one hundred calculating machines, including one which is thought to be the first of its kind. He wrote a "History of Mathematics," published in 1916; and in 1923 he published a volume called "Ancient Quipu" describing the primitive counting system used by early Peruvians. He was sixty-two years old.

Professor Henry Lewis Rietz, Professor of Mathematics at the University of Iowa, died on December 7, 1943. He retired as head of the department in 1942. He was president of the Mathematical Association of America in 1924 and was vice-president of the American Statistical Association in 1925, president of the Iowa Academy of Science in 1931 and was first national president of the Institute of Mathematical Statistics following its organization in 1935. He was also a member of the National Council of Teachers of Mathematics.

Professor Rietz was a member of Alpha Tau Omega, social fraternity; Sigma Xi, honorary scientific fraternity; Phi Beta Kappa, National Scholastic fraternity; and Gamma Alpha, honorary graduate scientific fraternity.

He was the principal author of a number of college texts in mathematics which were remarkably successful. In addition to books and articles he published over 150 significant articles in various journals.

At the third meeting of the Men's Mathematics Club of Chicago and the Metropolitan Area on December 17, 1943, Mr. David Rappaport of Lane Technical High School led a discussion on "Helpful Hints for Learning and Teaching Mathematics." Professor Zens L. Smith of the Institute of Military Studies at the University of Chicago, spoke on the topic, "Mathematics in the Military Program."

At the fourth meeting of the above club Mr. Nels A. Baird, Assistant Engineering Manager of the Douglas Aircraft Company, Inc., at the Chicago Plant, spoke on the topic, "Design for Flying" and "The B-19." Mr. Oliver Lee, Director of Dearborn Observatory, at Northwestern University, also spoke on the topic, "Dynamics and Dynamic Fossils in the Solar System."

The annual joint dinner meeting of the Men's Mathematics Club of Chicago and Metropolitan Area and the Women's Mathematics Club of Chicago and Vicinity was held on Friday, February 11, 1944. Professor Walter W. Hart, nationally known teacher and textbook writer, was the guest speaker. His topic was, "Post-War Mathematics Teaching."

Dr. Forrest E. Long, chairman of the Department of Secondary Education at New York University, has been named director of the School and College Division of the National Safety Council. He succeeds Miss Marian L. Telford, who has moved to the Council's Field Organization Department.

In his new capacity Dr. Long assumes active direction of the Council's expanded program among schools and colleges.

Dr. Long brings to his new position many years' experience in various phases of education. From 1915 to 1917 he taught high school mathematics and science in Missouri and Kansas. Upon his discharge from the Army following World War I, he became principal of the Junior-Senior High School at Atchison, Kansas. Dr. Long was made head of the Department of Secondary Education at the University of Tennessee in 1923. In 1924 he went to Washington University in St. Louis as associate professor of education.

Forrest E. Long

In 1928 he joined the staff of the New York University School of Education. He became acting chairman of the Department of Secondary Education in 1940, and chairman in 1941.

Dr. Long is the author of several textbooks on secondary education. Since 1929 he has served as editor of *The Clearing House*. He has been active both in the National Education Association and the Progressive Education Association, and has served both as director and as treasurer of the latter organization.

He holds the degrees of A.B., from William Jewell College, 1917; M.Ed., from Harvard University, 1923; and Ph.D., from New York University, 1928. He is a member of Phi Delta Kappa, Kappa Delta Pi, Kappa Phi Kappa and Phi Gamma Delta.

The annual meeting of the California Mathematics Council was held on December 28–29, 1943 at the Administration Building in Oakland, California.

Program

Tuesday, December 28—9:45 A.M.

General Meeting—Mathematics: Immediate Needs and Long Term Goals

Chairman: Lucian B. Kinney, Associate Professor and Acting Dean of School of Education, Stanford University and Executive Secretary C.M.C.

Panel: Hubert Armstrong, Director of Research, Oakland Public Schools and M. E. Muchlitz, Deputy Superintendent of Schools, Ventury County, Co-chairman C.M.C. Statewide Committee of Research in Mathematics and members of committee who are able to be here.

1:30 P.M. Section Meetings

1. Algebra and Advanced Mathematics

   Chairman: Dr. Harold Bacon, Associate Professor of Mathematics, Stanford University

Panel: Miss Florence Flanagan, San Jose State College and Secretary C.M.C. Miss Harriette Burr, Ventury Junior High and Treasurer C.M.C. Miss Emma Hesse, Claremont Junior High School. Mr. James Hoge, University High School. Miss Edith Eaton, Oakland High School.

2. Arithmetic

   Chairman: Mr. Charles C. Grover, Principal Glenview Elementary School and Advisor of C.M.C.

Speakers: Mr. M. E. Mushlitz, *Diagnosis in the Teaching of Arithmetic* and Mr. Hubert C. Armstrong, *Importance of Vocabulary in Mathematics*

3. Science of Aeronautics

   Chairman: Cornelius H. Siemens, Associate Professor of Education, University of California and Educational Consultant, C.A.A.

3:45 P.M. Business meeting of C.M.C.

7:30 P.M. General meeting

Chairman: Mrs. Ruth G. Sumner, Oakland High School and President C.M.C.

Speaker: Frank Freeman, Dean of School of Education, University of California

Subject: *Mathematics for the Millions*

Wednesday, December 29—9:45 A.M.

Section Meetings

1. *Arithmetic*

   Mr. Grover, presiding

   Some Recent Data with Regard to Bridging in Subtraction, Mrs. Marguerite Brydegaard, Assistant Professor of Education at San Diego State College

2. *Geometry*

   Miss Hesse, presiding

   Visual Aids, Rachel Keniston, Stockton High School

   Geometry in Art, Hamilton Wolff, California, School of Arts and Crafts

3. *Science of Aeronautics*

   Dr. Siemens, presiding

An Evaluation Curriculum

Panel: Adren Aiken, Mill Valley High School; Mauri Gould, Albany High School; Lawrence Hill, San Jose High School; A. L. Jordan, Polytechnic High School, San Francisco; E. W. Long, Technical High School, Oakland; Clarence Nelson, Hayward High School; Melvin Peterson, Lowell High School, San Francisco; Harry Sullivan, United Air Lines; and John Urlanb, Berkeley High School.

1:30 P.M.

General Meeting—Summary of Conference

Chairman: Professor Chester Jaeger, Dean of Department of Mathematics, Pomona College and Advisor of C.M.C.

Panel members selected by sections

Committee on Arrangements

Program: Mrs. Ruth Sumner and Section Chairmen
Publicity: Miss Edith Eaton
Housing: Mrs. C. McHew
Building Arrangements: James Hoge
Registration: Miss Flanagan and Miss Burr

Joint Meeting

NATIONAL AND INDIANA COUNCILS

Teachers of High School Mathematics

Indiana War Memorial Building

Indianapolis, Indiana

April 15, 1944

President—A. Dale Allen, New Castle H.S., New Castle

Vice-President—R. M. Takala, University School, Bloomington

Secretary-Treasurer—Miss Catherine M. Bennett, Concannon High School, West Terre Haute

Morning Session—9:30–11:45

1. Reading of the minutes
2. "How We Can Sell Mathematics to High School Pupils"
   Leader—Curtis D. Kirklin, Franklin College
   General Discussion
3. "National Mathematics Problems as Discussed at the National Council Meeting in New York City"
   Leader—L. H. Whitcraft, Ball State Teachers College
   General Discussion
4. "Research in Arithmetic Improvement of Seniors"
   Leader—Albert R. Mahin, Hartford City
   General Discussion
5. "Evaluation of New Mathematics Texts Approved in Indiana"
   Leader—Inez Morris, Indiana State Teachers College
   General Discussion
6. Appointment of Nominating Committee

Noon Intermission

Afternoon Session—1:30–3:30

7. Report of the Nominating Committee
8. "What We Owe to Pupils Of High Ability in Mathematics"
   Leader—L. M. Keisling, Kokomo
   General Discussion
9. "High School Mathematics After the War"
   Leader—Walter H. Carnahan, State Department of Education
   General Discussion
10. "Common Errors in Mathematics and How to Eliminate Them"
    Leader—Harry C. Snyder, New Castle
    General Discussion
11. "What is the Place of Approximate Computation in the High School Mathematics Curriculum?"
    Leaders—R. M. Takala and Philip Peak, University School, Bloomington
12. Adjournment

MATHEMATICAL ASSOCIATION OF AMERICA

Third Annual Meeting
Metropolitan New York Section
New York University, Saturday, April 22, 1944

Program

Morning Session, 9:30 A.M., Room 703, Main Building

Chairman: Professor R. M. Foster, Polytechnic Institute of Brooklyn

Address of Welcome:
Dean Charles M. McConn, Washington Square College, New York University

Elementary Mathematical Theory of External Ballistics:
Professor Harris F. MacNeish, Brooklyn College

Applications of Mathematics in Aeronautical Engineering:
Professor R. Paul Harrington, Polytechnic Institute of Brooklyn

Combinatorial Statistics:
Dr. Jacob Wolfowitz, Columbia University

Afternoon Session, 2:00 P.M., Room 703, Main Building

Chairman: Mr. Max Peters, Long Island City High School

Business Meeting and Election of Officers for 1944–1945

A Guiding Philosophy for Teaching Demonstrative Geometry:
Mr. Morris Hertzig, Forest Hills High School

Mathematics and Empirical Science:
Professor Carl G. Hempel, Queens College

Officers of the Metropolitan New York Section, 1943–1944

Ronald M. Foster, Polytechnic Institute of Brooklyn.......................Chairman
Max Peters, Long Island City High School ....Vice-Chairman
Howard E. Wahlert, New York University ....Secretary
Frederic H. Miller, Cooper Union....Treasurer